Exploration of the Equations, Graphics, Prerequisites, and Sales Range of Reasonable Yield Curves from the Consumer’s Perspective

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ABSTRACT

This study first evaluates previous theories of the yield curve and the error incurred in the associated graphics and equations from the marginal utility, the price willingly paid by the consumers, and reasonable consumption range acquired by merely considering from the producer’s perspective while neglecting the various consumption amounts of the consumers. Moreover, this study explores the reasonable yield curve, its equations, graphics, required prerequisites, seven sales ranges, and reasonable sales range and verifies whether the equations are reasonable and valid by considering from the consumer’s perspective in an imperfectly competitive market with a focus on marketing orientation, returning to the need aspect. At the end, the results of the study are presented in graphics and tables are generated to compare the individual key points in the AR and MR curves, the corresponding sales quantities, and the required prerequisites.

INTRODUCTION

Motivation and Purposes

A review of previous literatures regarding the theories of yield curves reveals that all existing theories explore the relations of AR, TR, MR, and Ed acquired for different sales quantities from the perspective of the producers (Lu, 2005). However, such study is not need-oriented to explore various sales quantities, MU acquired by the consumers, and the price willingly (or actually paid) by the consumers from the perspective of the consumers as the basis of production and sales (Shieh, 2011). In addition, in the past, the aforementioned theories explored the situation where the sales quantity cannot exceed the satiation point, MR-0, maximum TR, and Ed=1 merely from the viewpoint of the producers. It does not explore from the viewpoint of the consumers, exploring the maximum reasonably needed amount and minimum reasonably needed amount as the basis for sales (Milgrom & Roberts, 1992).

Moreover, the hypothesize of the above theories is that the AR curve is a curve extending from upper left to lower right without considering LDMU, the marginal utility of consuming a single good may progressively increase or remain unchanged in an extremely short period of time. In other words, the price willingly paid or actually paid by the consumers within a very short period of time may progressively increase or remain unchanged, leading to a reasonable result of an AR curve extending from lower left to upper right and reaching a leveled stage. Furthermore, the theories assumed a positive intercept between the AR curve and the vertical axis, which implies an unrealistic situation where the highest AR is obtained when the sales quantity is zero. The AR curve and the associated equations
deduced following the above theories apparently miss to consider the realistic situations and should be treated as unreasonable and incorrect (Riddals & Bennett, 2001).

Lastly, hardly any of the previous theories differentiates various sales ranges according to the associated sales quantities, let alone exploring the prerequisites required by each individual range, reasonable sales range, individual minimum and maximum sales quantities as the basis for sales (Li, 1993).

In summary, this study explores the reasonable yield curve and its equations, graphics, prerequisites, seven sales ranges, and reasonable sales ranges; verifies whether the equations are reasonable and valid by being marketing-oriented; returns to the need aspect, assuming the viewpoint of consumers in an imperfectly competitive market (Jensen & Meckling, 1976). The purpose of this study is described in details below:

1. With consideration from the perspective of the consumers along with an orientation on need, explore the MU and the price willingly paid by the consumers in different sales quantities as the basis of AR, TR, MR and $E_d$ acquired by producers.
2. Based on LDMU, present fully realistic graphics generated from the relations of AR, MR, TR curves.
3. List equations of reasonable AR, MR, and TR curves and verify their validity.
4. Differentiate seven sales ranges and explores the AR and MR acquired by the producers, and the associated prerequisites and sales quantities based on LDMU and by the capacity of consumption of the consumers.
5. Find the maximum and minimum reasonable sales quantities.

**Research flow / Steps and basic Hypotheses**

**Research flow / Steps**

1. Propose the basic Hypotheses.
2. Based on LDMU, plot the corresponding graphics and generate tables to compare the required conditional equations and graphics of reasonable AR and MR curves in the seven sales ranges.
3. List the equations for the reasonable AR, MR, and TR curves and verify whether they are reasonable and valid.
4. Plot and compare by tables the inflection points of the AR and MR curves, the highest point, the horizontal intercept, and the associated required conditional equations and sales quantities.
5. Describe reasonable sales ranges and the corresponding individual sales quantities.

**BASIC HYPOTHESES**

Based on the aforementioned research purpose and flow, this study proposes the following basic hypotheses:

**Hypothesis 1 (H1):** Consumer spends all his/her income on purchasing single goods.

**Hypothesis 2 (H2):** According to the Cardinal Utility Analysis by Marshall, MU remains unchanged.

**Hypothesis 3 (H3):** Consumer uses the MU acquired in the consumption as the highest price willingly paid or actually paid.

**Hypothesis 4 (H4):** LDMU is valid, meaning that within a certain period of time when the consumer purchases a specific goods, MU decreases with increasing consumption amount. Undeniably, however, MU may also progressively increase or remain unchanged within a short period of time.

**Hypothesis 5 (H5):** When the consumption amount is zero, MU equals to zero and the price willingly paid (or actually paid) by the consumers is zero as well.
Hypothesis 6 (H6): Starting from the origin, the MU curve first moves at a progressively increasing speed, followed by a progressively decreasing speed, where the speed starts decreasing progressively after it reaches the maximum.

Hypothesis 7 (H7): The MU curve and AR curve are of the same type.

Types of reasonable AR curve and comparison of seven sales ranges

In this section, combined with Fig. 1 and Table 1, seven sales ranges are defined according to the sales amount. A brief analysis of the required equations with restrictions and the implied meaning for the consumer behavior (Pei-Wen Jing, 2011) is given below:

First Range (between \( OQ_1 \))

When the sales quantity is zero, the price willingly paid (or actually paid) by the consumers should be zero. As the sales quantity increases, \( AR \) increases progressively. The equation must satisfy the following three restrictions: the price willingly paid by the consumers is higher than or equal to zero, the slope of \( AR \) curve is positive, and the slope is increasing.

Second Range (\( Q = Q_1 \))

As the sales amount increases, the speed at which \( AR \) increases remains unchanged, leaving \( AR \) at the inflection point of the curve. The equation must satisfy the following three restrictions: the price willingly paid by the consumers is higher than zero, slope of \( AR \) curve is positive, and the slope remains the same.

Third Range (between \( Q_1Q_2 \))

As the sales amount increases, \( AR \) increases with decreasing speed. The equation must satisfy the following three restrictions: the price willingly paid by the consumers is higher than zero, slope of \( AR \) curve is positive, and the slope progressively decreases.

Fourth Range (\( Q = Q_3 \))

In this range, the \( AR \) curve reaches its maximum (slope is zero) and the price willingly paid by the consumers remains unchanged when the sales amount changes. The equation must satisfy the following three restrictions: the consumers are willing to pay the highest price, the slope of \( AR \) curve is zero, and the slope decreases progressively.

Fifth Range (between \( Q_2Q_3 \))

As the sales amount increases, \( AR \) decreases progressively and is still higher than zero, with the curve extending from upper left towards lower right (slope is negative) and staying above the horizontal axis. This sales range fits the descriptions of LDMU. Before (including) this range, \( MU > 0 \), the associated type of goods is Preferred Goods. The equation must satisfy the following three restrictions: the price willingly paid by the consumers is higher than zero, the slope of \( AR \) curve is negative, and the slope decreases progressively.

Sixth Range (\( Q = Q_3 \))

In this range, the \( AR \) curve extends from upper left towards lower right (slope is negative) and crosses the horizontal axis with maximum \( TR, MR = 0 \), and \( E_d = 1 \) The Satiation Point of sales is reached.
with $MU=0$ and the associated type of goods is Independent Goods. The equation must satisfy the following three restrictions: the price willingly paid by the consumers equals to zero, the slope of $AR$ curve is negative, and the slope decreases progressively.

**Seventh Range (beyond $Q_3$)**

As the sales amount keeps increasing, the $AR$ curve extends from upper left towards lower right (slope is negative) and falls below the horizontal axis. $MU<0$ and the associated type of goods is Disgust Goods, where the consumers are willing to continue with the consumption only when getting paid. The equation must satisfy the following three restrictions: the price willingly paid by the consumers is lower than zero, the slope of $AR$ curve is negative, and the slope decreases progressively (Wu, 2012).

![Type of the reasonable AR curve and the seven sales ranges.](image)

**Note:**
- Phase I shows $MU$ increases in a gradual speed;
- Phase II shows $MU$ decreases in a gradual speed

**Table 1: Comparison of required restrictions for the seven sales ranges.**

<table>
<thead>
<tr>
<th>Prerequisites</th>
<th>Range</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $AR$</td>
<td></td>
<td>$\geq0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>Maximum</td>
<td>$&gt;0$</td>
<td>$=0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>2. $\frac{dAR}{dQ}$</td>
<td></td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$=0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>3. $\frac{d^2AR}{dQ^2}$</td>
<td></td>
<td>$&gt;0$</td>
<td>$=0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
</tr>
</tbody>
</table>
Equations of $AR$ and $MR$ curves, comparison of each key point, and empirical verification of reasonable sales ranges.

Equations of $AR$, $TR$, $MR$ and their verification

Equations of $AR$, $TR$, and $MR$

$AR = P = aQ + 2i\sqrt{ac}Q^3 - cQ^3 \quad , \quad a > 0, c > 0 \quad \tag{1}$

$TR = aQ^2 + 2i\sqrt{ac}Q^3 - cQ^4 \quad \tag{2}$

$MR = 2aQ + 6i\sqrt{ac}Q^2 - 4cQ^3 \quad \tag{3}$

$P$: Price
$Q$: Quantity

Verify: When $TR$ reaches its maximum, $E_d = \frac{dQ}{dP} \frac{P}{Q} = 1, \quad MR = 0$

$E_d = \frac{dQ}{dP} \frac{P}{Q} = 1$

According to Eq. (1),

$$\Rightarrow \frac{1}{a + 4i\sqrt{ac}Q - 3cQ^2} \frac{aQ + 2i\sqrt{ac}Q^3 - cQ^3}{Q} = 1,$$

$$\Rightarrow Q^* = i\sqrt[3]{\frac{a}{c}} \quad \tag{4}$$

$MR = 0$

According to Eq. (3),

$$\Rightarrow Q^* = \frac{3 + 1}{4} i\sqrt[3]{\frac{a}{c}},$$

$$\Rightarrow Q^* = i\sqrt[3]{\frac{a}{c}}, \quad (Q^* = \frac{i}{2} \sqrt[3]{\frac{a}{c}}, \text{which is unreasonable}) \quad \tag{5}$$

Eq. (4) = Eq. (5), indicating that the hypothesis for the above equations are reasonable and valid.

Comparison of key points of $AR$ and $MR$ curves, as shown in Fig. 2 and Table 2 $AR$ Curve

Inflection point (point A): $Q = Q_1$, and Eqs. (6), (7), and (8) below must be satisfied.

$AR = aQ + 2i\sqrt{ac}Q^3 - cQ^3 > 0 \quad \tag{6}$

$\frac{dAR}{dQ} = a + 4i\sqrt{ac}Q - 3cQ^2 > 0 \quad \tag{7}$

$\frac{d^2AR}{dQ^2} = 4i\sqrt{ac} - 6cQ = 0 \quad \tag{8}$

According to Eq. (8),

$$\Rightarrow Q = Q_1 = \frac{2i}{3} \sqrt[3]{\frac{a}{c}}$$
Highest point (point B): \( Q = Q_2 > Q_1 \), and Eqs. (9), (10), and (11) below must be satisfied.

\[
\begin{align*}
AR & > 0 \hspace{1cm} \text{(9)} \\
d\frac{AR}{dQ} & = 0 \hspace{1cm} \text{(10)} \\
d^2\frac{AR}{dQ^2} & < 0 \hspace{1cm} \text{(11)}
\end{align*}
\]

According to Eq. (10),
\[
\Rightarrow Q = Q_2 = \frac{2 \pm 1}{3} \sqrt[3]{\frac{a}{c}} ,
\]
\[
\Rightarrow Q_2 = i \sqrt[3]{\frac{a}{c}} , \hspace{0.5cm} \text{(if} \quad Q_2 = \frac{i}{3} \sqrt[3]{\frac{a}{c}} < Q_1 = \frac{2i}{3} \sqrt[3]{\frac{a}{c}} \text{, then the result is unreasonable.)}
\]

Horizontal intercept (point C): \( Q = Q_3 > Q_2 > Q_1 \), and Eqs. (12), (13), and (14) below must be satisfied.

\[
\begin{align*}
AR & = 0 \hspace{1cm} \text{(12)} \\
d\frac{AR}{dQ} & < 0 \hspace{1cm} \text{(13)} \\
d^2\frac{AR}{dQ^2} & < 0 \hspace{1cm} \text{(14)}
\end{align*}
\]

According to Eq. (12),
\[
\Rightarrow Q = Q_3 = 2i \sqrt[3]{\frac{a}{c}}
\]

MR Curve

Inflection point (point F): \( Q = Q_6 \), and Eqs. (15), (16), and (17) must be satisfied.

\[
\begin{align*}
MR & = 2aQ + 6i\sqrt{ac}Q^2 - 4cQ^3 > 0 \hspace{1cm} \text{(15)} \\
d\frac{MR}{dQ} & = 2a + 12i\sqrt{ac}Q - 12cQ^2 > 0 \hspace{1cm} \text{(16)} \\
d^2\frac{MR}{dQ^2} & = 12i\sqrt{ac} - 24cQ = 0 \hspace{1cm} \text{(17)}
\end{align*}
\]

According to Eq. (17),
\[
\Rightarrow Q = Q_4 = \frac{i}{2} \sqrt[3]{\frac{a}{c}}
\]

Highest point (point G): \( Q = Q_5 > Q_6 \), and Eqs. (18), (19), and (20) below must be satisfied.

\[
\begin{align*}
MR & > 0 \hspace{1cm} \text{(18)} \\
d\frac{MR}{dQ} & = 0 \hspace{1cm} \text{(19)} \\
d^2\frac{MR}{dQ^2} & < 0 \hspace{1cm} \text{(20)}
\end{align*}
\]

According to Eq. (19),
\[
\Rightarrow Q = \frac{3 \pm \sqrt{3}}{6} i \sqrt[3]{\frac{a}{c}} ,
\]
\[ Q = Q_s = \frac{(3 + \sqrt{3})i}{6} \sqrt{\frac{a}{c}}, \text{ (if } Q_s = \frac{(3 - \sqrt{3})i}{6} \sqrt{\frac{a}{c}} < Q_4, \text{ then the result is unreasonable.)} \]

**Horizontal intercept (point H):**  \( Q_o > Q_s > Q_4 \) and Eqs. (21), (22), and (23) below must be satisfied.

\[
\frac{dMR}{dQ} < 0 \quad \text{..............................................................(21)}
\]

\[
\frac{d^2MR}{dQ^2} < 0 \quad \text{..............................................................(22)}
\]

According to Eq. (21),

\[ Q = \frac{3 \pm 1}{4} i \sqrt{\frac{a}{c}}, \]

\[ Q = Q_o = i \sqrt{\frac{a}{c}}, \text{ (if } Q_o = \frac{i}{2} \sqrt{\frac{a}{c}} = Q_4, \text{ then the result is unreasonable.)} \]

---

**Fig 2: Relation of reasonable AR and MR curves.**

<table>
<thead>
<tr>
<th>Properties and Quantity</th>
<th>Location of point</th>
<th>Inflection point</th>
<th>Highest point</th>
<th>Horizontal intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. AR, MR</td>
<td></td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>= 0</td>
</tr>
<tr>
<td>2. ( \frac{dAR}{dQ}, \frac{dMR}{dQ} )</td>
<td></td>
<td>&gt; 0</td>
<td>= 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>3. ( \frac{d^2AR}{dQ^2}, \frac{d^2MR}{dQ^2} )</td>
<td></td>
<td>= 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>4. ( Q ) of AR =</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( Q_1 = \frac{2i}{3} \sqrt{\frac{a}{c}} )</td>
<td>( Q_2 = i \sqrt{\frac{a}{c}} )</td>
<td>( Q_3 = 2i \sqrt{\frac{a}{c}} )</td>
</tr>
<tr>
<td>5. ( Q ) of AR =</td>
<td></td>
<td>( Q_1 = \frac{i}{2} \sqrt{\frac{a}{c}} )</td>
<td>( Q_2 = (3 + \sqrt{3})i \sqrt{\frac{a}{c}} )</td>
<td>( Q_6 = i \sqrt{\frac{a}{c}} )</td>
</tr>
</tbody>
</table>
Reasonable Sales Ranges $Q_2Q_3$

Since the reasonable price and consumption capacity paid by the consumers should be higher than zero, the reasonable AR curves will exist only in the first quadrant, namely between the first and fifth ranges. If the LDMU requirement must also be satisfied, the AR curve should extend from upper left towards lower right (slope is negative). The corresponding reasonable sales range should exist only in the area where the highest point and the horizontal intercept of the AR curve intersect with each other, i.e., the fifth range (between $Q_2Q_3$). In other words, it is in the range between $Q=Q_2$ for the minimum reasonable sales amount and $Q_2=Q_3$ for the maximum reasonable sales amount (Cliffs Notes, 2012).

CONCLUSION

As a summary, the contribution in research, practical contribution, and the direction of follow-up research subjectively identified by this study are described below.

Contribution in research

This study evaluates the previous yield curves for ignoring LDMU and theoretical flaws in the associated reasonable sales ranges and equations with restrictions.

Based on LDMU, this study defines seven sales ranges to explain the meaning of the consumer behavior.

In the application and innovation of theories, this study uses graphics and tables to compare the reasonable yield curve, basic theoretical hypotheses, various sales ranges, types of curves, equations, and equations with restrictions.

In addition, this study uses graphics and tables to compare the properties of AR and MR curves, individual key points, and reasonable sales ranges, which represents the application and innovation of this study in related theories.

Practical contribution

This study can provide the producers with the sales ranges identified by the consumers for understanding the preferences of the consumers (price willingly paid) to offer the optimum pricing and establish suitable marketing strategies.

This study can also provide relevant information to the producers to control the production within the reasonable sales range for improved efficiency in resource utilization.

Follow-up direction of research/hypothesis

In hypothesis, $MU_M$ remains unchanged. However, the actual $MU_M$ should decrease progressively, which will change the vertical axis in the graph of the AR curve in proportion to the changes in $MU_M$, for example enlarging ($MU_M > 1$) or reducing ($MU_M < 1$).

This study is based on the hypothesis that the consumer spends all his/her income on purchasing single goods; the study does not discuss savings and balancing multiple types of goods.

Any interested researchers in the future can continue with further study of the theories of the yield curve according to the two hypotheses above.
REFERENCES


