Bond Prices and Interest Rates: Techniques for Teaching Their Inverse Relationship

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ABSTRACT

The paper addresses the pedagogy involved in teaching the inverse relationship between bond prices and interest rates. After reviewing the techniques for explaining this relationship employed in popular textbooks on financial markets, three approaches from somewhat atypical perspectives are offered. The approaches emphasize the fixity of coupon payments, the arithmetic of fractions and the simultaneous supply and demand in the markets for bonds and loanable funds. After presenting the three approaches in actual classroom settings, quizzes were administered and empirical results are reported on the relative effectiveness of the alternative approaches. Results suggest that particular approaches can indeed make a difference in students' understanding of this key financial markets relationship and also suggest possible diminishing returns as the number of different explanations of a given concept increases.

INTRODUCTION

There's no getting away from it—students taking courses in Finance will run into the inverse relationship between bond prices and interest rates, a relationship described as "…one of the most fundamental relationships in finance" according to one popular text. [Saunders and Cornett, p. 31] Whether the primary focus is financial markets, corporate finance, or investments, coverage of the bond-price-interest-rate nexus is a given. While few students may ever end up using other details of bond pricing, at a minimum all students should be aware of and understand this inverse relationship. In this paper, we review the explanations provided in leading texts, present three approaches that employ slightly different perspectives for explaining the inverse relationship between bond prices and interest rates and finally report quantitative results from the application of the different approaches in the classroom.

EXPLANATIONS FROM LEADING TEXTS

A review of popular undergraduate textbooks on financial markets reveals a variety of explanations of the inverse relationship between bond prices and interest rates.

Mishkin and Eakins (2009) in their chapter, "What Do Interest Rates Mean and What Is Their Role in Valuation?", for example, state that, "It is straightforward to show that the valuation of a bond and the yield to maturity are negatively related. As *i*, the yield to maturity, rises, all denominators in the bond pricing formula must necessarily rise. Hence a rise in the interest rate as measured by the yield to maturity means that the value and hence the price of the bond must fall. Another way to explain why the bond price falls when the interest rises is that a higher interest rate implies that the future coupon payments and final payment are worth less when discounted back to the present; hence the price of the bond must be lower."[p.49] While the foregoing is certainly correct, it can be less than illuminating for students being exposed to the concept for the first time and particularly so for those students who are quantitatively challenged. In order for the first explanation to enhance student understanding requires a good grasp of basic arithmetic—something lacking in many undergraduates today. In a similar vein, the second explanation requires basic understanding of present value, a concept not necessarily mastered by students in introductory financial markets classes. Aside from these concerns, and probably because of the target audience, neither of the explanations is accompanied by an illustration of the points being made.

In the following chapter, "Why Do Interest Rates Change?" all of the discussion is in terms of supply and demand for bonds and hence the equilibrium bond price with only a reference to the inverse impact on the level of the interest rate. In the authors' discussion and graphical illustration of the Fisher Effect, for example, shifts in supply and demand for bonds result in a reduction of the equilibrium bond price. The accompanying impact on the interest rate is "explained" by the following: "The equilibrium bond price has fallen from P1 to P2, and because the bond price is negatively related to the interest rate, this means that the interest rate has risen." [p.86]

In earlier editions of this same text, the authors employed graphical illustrations of the market for bonds with the bond price measured in the typical way on the vertical axis but with the interest rate measured on a right-hand vertical axis and moving inversely to bond prices. Rather than aiding the student in understanding the inverse relationship, this simply asserts it. It's interesting to note that in the Preface to the 6th edition, Mishkin and Eakins (2009) indicate under the heading of "Improved Exposition and Organization" that as a result of helpful comments from reviewers that they decided to "…simplify the exposition …by eliminating the right-hand axis on the graphs, which had interest rates going in the wrong direction. The analysis now focuses on what happens to bond prices and emphasizes that when bond prices rise, interest rates fall , and vice versa".

Madura (2008) presents examples of bond prices at various discount rates but never emphasizes the inverse relationship between these two variables let alone provides any explanation other than whatever the reader can surmise from the results in the examples themselves. Moreover, while Madura employs the loanable funds framework to explain the determination of interest rates, he never ties the results he illustrates to bond prices, either graphically or in the text.

Fabozzi, Modigliani and Jones (2010) in their chapter, "Properties and Pricing of Financial Assets", likewise use an example of a combination of bond prices and discount rates as the core of their coverage of the inverse relationship between these two variables. The accompanying "explanation" consists of the following: "…it should be clear that the price of a financial asset changes as the appropriate discount rate, r, changes. More specifically, the price changes in the opposite direction to the change in the appropriate discount rate. An illustration of this principle appears in Table 9 – 1, which shows the price of our hypothetical financial asset for various discount rates." [p.177]

Saunders and Cornett (2009) in their chapter, "Determinants of Interest Rates", note that, "The inverse relationship between the value of a financial instrument—for example, a bond—and interest rates is one of the most fundamental relationships in finance, and is evident in the swings that occur in financial asset prices whenever major changes in interest rates arise." [p.31] Their initial explanation then rests on observed combinations of present values and discount rates based on a simple lump-sum present value calculation. Their explanation utilizes the common present value formulation as they note that the reason for the observed changes in opposite directions of discount rates and present values is "…because as interest rates increase, fewer funds need to be invested at the beginning of an investment horizon to receive a stated amount at the end of the investment horizon." [p.31] In the following chapter, "Interest Rates and Security Valuation", the authors provide an example of a bond's price at three different discount rates and then "illustrate" this relationship with a plot of the combinations.

In a treatment notable for more in-depth graphical analysis, Hubbard (2008) in the chapter, "Interest Rates and Rates of Return", states, "This inverse relationship between the yield to maturity and the price of the instrument is important, and the reason for it can be explained this way: discounting future payments at a higher rate necessarily reduces the present value of the payments and hence the value or price of the bond. A lower yield to maturity raises the present value of the future payments and hence the price of the bond." [p. 72] Once again, understanding of this explanation requires a solid understanding of the present value concept. Hubbard goes further than other authors, however, in providing a detailed graphical explanation for the inverse relationship between bond prices and interest rates. In a subsequent chapter, "Determining Market Interest Rates", Hubbard presents both sides of the debt transaction market—that for bonds and that for loanable funds—and illustrates his point with graphs for each market. This approach emphasizes the fact that the demand for (supply of) bonds can also be viewed as the supply of (demand for) loanable funds. This approach effectively utilizes and extends the earlier approach of Mishkin and Eakins (2009) of graphically illustrating the simultaneous changes in bond prices and market interest rates.

With few exceptions, a review of popular undergraduate texts on financial markets reveals that the dominant explanation for the inverse relationship between bond prices and interest rates is essentially a present-value explanation. For students who have not yet mastered Time Value of Money or who are quantitatively challenged, this approach may not constitute an "explanation" at all. While any formulaic explanation must rely on the present value framework, descriptions of this relationship from slightly different perspectives can drive home the same point without requiring that the student has mastered Time Value of Money techniques. In the following section, we describe approaches that utilize such slightly different perspectives.

Approach 1: When Payments Are Fixed Price Must Carry the Burden

We begin with a simple investment of a perpetuity and pose the following: if this security promises an annual payment of $10 forever and if the investor wants to earn 10%, what is the most that the investor should be willing to invest in this security? In other words, $10 a year would represent 10% of what invested amount? Even the least quantitatively-oriented students can usually come up with the correct answer of $100 pretty quickly. We then suppose that for whatever reason(s), market interest rates or the required rate of return increases to 20% and we consider two different types of bonds. One is a fixed-price type and the other is a fixed-payment type. We then ask what would happen with each type of bond in order for the investor to earn a 20% rate of return. Students can usually determine rather quickly that for the fixed-price bond, the annual payment would need to increase to $20, whereas with the fixed-payment bond, the ***only*** way to give the investor the required rate of return is to give him the same $10 a year but require him to invest only $50. It's the fact that the future ***dollar return*** is fixed that means that the only way to alter the ***percentage return*** for the investor is to have the amount invested--the price of the bond --move in the opposite direction. All of this is illustrated in Table 1 where once again, most students can come up with the correct answer of $50.

Table 1

	Starting Figures	Fixed-Price Bond	Fixed-Payment Bond
Required Rate of Return	10%	20%	20%
Annual Payment	$10	$20	$10
Price of Bond	$100	$100	**??**

Approach 2: It's Simply A Matter of Arithmetic

In order to avoid long tedious calculations, students are often introduced to a bond pricing formula that takes advantage of the formula for the present value of an annuity. In particular, the formula for the price of a bond can be presented as the present value of an annuity (the fixed coupon payments) plus the present value of a lump sum as in (1) below.

$P_B = C[(1-1/(1+r)^t)/r] + M/(1 + r)^t$ (1)

where: P_B = price of the bond

C = periodic coupon payment

r = required rate of return

t = number of payments

M = maturity value

For purposes of helping students understand the inverse relationship between bond prices and interest rates, however, it is helpful to present the bond pricing formula as the sum of fractions that represent each individual payment. Expression (2) does this for a one-year annual coupon bond where C_1 is the first year's annual coupon payment, M is the maturity or par value and r is the market rate of interest for an investment of this maturity and risk.

$P_B = C_1/(1+r)^1 + M/(1 + r)^1$ (2)

The key for student understanding with this approach is the fact that *as the numerator of a fraction is held constant, the value of the whole fraction will move in the opposite direction from the change in the denominator.* This is easily illustrated by the use of fractions that students can readily convert to decimals. It should be quite clear, for example, that as we go from ¼ to ½ , the value of the fraction increases from .25 to .50. This is an example of a fixed numerator and the value of the whole fraction moving in the opposite direction from the change in the denominator. On the heels of this example, students can then be directed to view the price of a bond as nothing more than the sum of a series of fractions whose numerators are fixed and whose denominators change with changes in interest rates. It doesn't matter what the specific values for the numerators are as the important feature is that those values are fixed for the life of the bond. Whatever those values are, as the denominator--the interest rate--changes, the value of each of the fractions will change in the opposite direction hence causing the value or price of the bond to also change in the opposite direction from the change in interest rates. Expression (3) illustrates the calculation for a one-year 10 percent annual coupon bond with the market interest rate at 8 percent.

$P_B = \$100/(1.08)^1 + \$1,000/(1.08)^2$ (3)

$P_B = \$92.59 + \$925.93 = \$1,018.52$

As the relevant interest rate increases to 10 percent we ask what has to happen to the values of the fractions in (3)

$P_B = \$92.59 + \$925.93 = \$1,018.52$

?? ?? ??

It can be seen in expression (4) that the values of both fractions making up the price of the bond decline leading to what has to be a lower price for the bond. The \$92.59 declines to \$90.91 and the \$925.93 to \$909.09 thereby reducing the price of the bond from \$1,018.52 to \$1,000.00.

$P_B = \$100/(1.10)^1 + \$1,000/(110)^2$ (4)

$P_B = \$90.91 + \$909.09 = \$1,000.00$

From this perspective, the inverse relationship between bond prices and interest rates is simply a matter of arithmetic. It should be noted as well, that this explanation doesn't require so much as a mention—let alone any understanding—of any explicit present–value framework. An understanding of the impact on the value of fractions with well-known decimal equivalents such as ¼ and ½ and a willingness to take as given the formula for the price of a bond is all that is required.

Approach 3: Demand and Supply Are Two Sides of the Same Coin

One very helpful device in teaching a variety of market-related topics is to emphasize to students that every demand carries with it a corresponding supply and vice versa. In other words, every demander is simultaneously a supplier and every supplier is simultaneously a demander. It is often helpful to ask students who might hold a job while going to school whether they are suppliers or demanders in the labor market. This question typically elicits the response of "supplier" which is then followed up with the question of just what it is that the student is supplying. Most tend to answer with "work", or the more terminologically sophisticated might respond with "labor services". The inquiry should not stop at this point, however, as an enlightening follow-up question of just what the student is demanding in return for their supply of labor services is a good way to jump-start the thought process along the lines of demand and supply being two sides of the same coin. A suggested image of a demander pounding his fist on a desk as his way of demanding something can also serve as a humorous entrée into the question of exactly what does a demander do in order to demand something?

Two foundation pieces are needed with this approach to teaching the inverse relationship between bond prices and interest rates. One is that a market participant is never simultaneously demanding and supplying the same asset and the other is that the supply and demand of the connected assets will always move in the same direction. Using a loanable funds framework, we would describe the two sides of the same coin as consisting of "bonds" on one side and "loanable funds" on the other. If, for example, we were to postulate an increase in the demand for bonds, we would emphasize that the way for an investor to ***demand*** bonds is to ***supply*** loanable funds. With this as a foundation, it is a short step to the conclusion that bond prices will rise and interest rates will fall.

We can illustrate this result using either descriptive relationships or with graphical analysis. In terms of descriptive relationships, consider the following:

(a) Interest rates represent a price—the rental price of money—and are affected by changes in supply and demand in precisely the same manner as is any other price, including the price of a bond; and

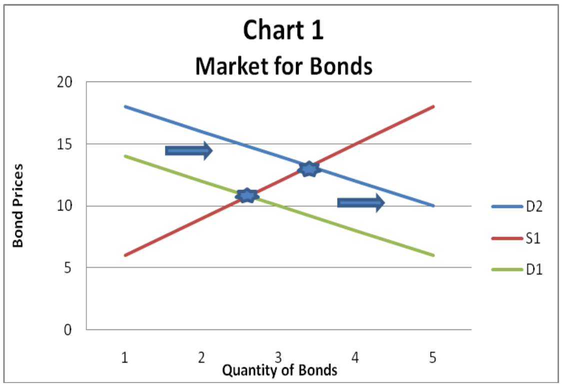
(b) Prices always change in the **same direction** as the **change in demand** and in the **opposite direction** from the **change in supply**

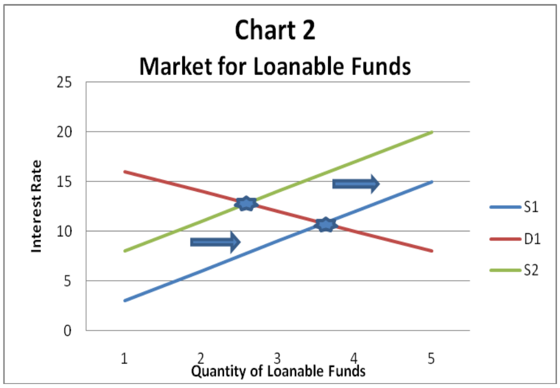
Table 2 below illustrates the derivation of the inverse relationship between bond prices and interest rates based on (a) and (b) above.

Table 2

Action:	↑ Demand for Bonds	=>	↑ Supply of Loanable Funds
Impact on Price (Interest Rate):	Increase (Same ↑)		Decrease (Opposite ↓)
Action:	↓ Demand for Bonds	=>	↓ Supply of Loanable Funds
Impact on Price (Interest Rate):	Decrease (Same↓)		Increase (Opposite↑)

This same result can be illustrated graphically as seen in Charts 1 and 2 below. In Chart 1 lenders increase their demand for bonds, resulting in upward pressure on bond prices. In Chart 2 the simultaneous flip side of investors' increase in the demand for bonds is the increase in their supply of loanable funds thereby placing downward pressure on interest rates.





In summary, the approaches above provide explanations for the inverse relationship between bond prices and interest rates utilizing slightly different perspectives from the more direct Time Value of Money framework commonly found in popular textbooks. In doing so the approaches described above emphasize the following:

(1) **Fixity of (Coupon) Payments:** Because (coupon) payments are fixed in dollars, the only way for the bond to yield the market rate of return is for its price to change in the opposite direction from the change in the interest rate.

(2) **Bond Price as Sum of Fractions with Fixed Numerators:** With constant numerators, changes in the denominator (the interest rate) will change the value of the all of the fractions in the opposite direction thereby changing the price of the bond in the opposite direction from the change in the interest rate.

(3) **Demanders (Suppliers) of Bonds are Simultaneously Suppliers (Demanders) of Loanable Funds and Prices of Bonds and Loanable Funds Move Directly with Changes in Demand and Inversely to Changes in Supply:** Changes in demand for (supply of) bonds must be accompanied by changes in supply of (demand for) loanable funds in the same direction causing bond prices and interest rates to change in opposite directions from each other.

In order to obtain a reading on the effectiveness of the approaches described above in helping students understand the inverse relationship between bond prices and interest rates, the explanations were presented to four different sections of various Finance classes during the fall 2009 semester at Temple University. We now present results of regression analysis designed to assess the effectiveness of the various explanations on student performance on a common quiz.

ESTIMATED EFFECTS OF ALTERNATIVE EXPLANATIONS

During the fall 2009 semester, the three approaches described above were presented to four different sections of various undergraduate Finance classes at Temple University. Sections involved were one each of "Introduction to Financial Markets" and "Financial Management" (introductory Corporate Finance) and two sections of a senior seminar in Financial Management which is the capstone course for Finance majors. Table 3 illustrates the alignment of the approaches and classes.

Table 3

Approach:	#1 Fixed Payments	#2 Fractions	#3 Joint Markets
Sections:			
Introduction to Financial Markets	X	X	X
Financial Management	X	X	
Senior Seminar: Section A	X		
Section B		X	

Presentations were made in each section and a 5-question quiz was then administered at some point in a later class meeting. In addition to scores on the quiz, data were gathered on the following student characteristics: (a) GPA (b) Non-quiz class performance [NQCP] (c) Gender (d) Ethnicity (e) Major and (f) Transfer status. Preliminary estimates indicated statistical significance for GPA and NQCP only with the latter exhibiting greater explanatory power than the former. As a result, subsequent estimates used NQCP as the key explanatory variable.

In an attempt to estimate the mean effects of the various approaches described above, dummy variables were used to represent one or more of the approaches employed to explain the inverse relationship between bond prices and interest rates. [See Rao and Miller, pp. 88-96]. Consider a specification such as equation (5) where the number of dummy variables included will be equal to one fewer than the number of approaches under study. A comparison of two approaches, for example, would require one dummy variable whereas a comparison of four would require three dummy variables.

$$Y = \beta_0 + \beta_1 X_1 + \alpha_1 D_1 + \varepsilon \qquad (5)$$

With the appropriate number of dummy variables, regression equation (5) can be used to estimate the mean effect of the various approaches on the corresponding impact of NQCP on quiz scores. The various approaches or combinations thereof are treated as qualitative variables that have a different mean effect for each class section. The estimated mean effect of the excluded approach—the one for which no dummy variable is included—is given by β_0 while the estimated mean effects of the included approaches are given by the sum of β_0 and the respective coefficients of the included dummy variables. For equation (5), for example, the mean effect of the approach represented by D_1 on X_1's impact on quiz scores would be equal to $(\beta_0 + \alpha_1)$. The dummy-variable specification was then used to address the following three questions:

(1) How do Approaches 1 and 2 compare?

(2) How does the combination of Approaches 1 and 2 compare to the combination of all 3 approaches?

(3) How do Approaches 1 and 2 alone compare to the combination of Approaches 1 and 2 and the combination of all 3 approaches?

For question (1), equation (5) was estimated with

β_0 = mean effect of Approach 2

X_1 = NQCP for the two senior seminar sections

D_1 = 1 for Approach 1 and 0 otherwise

and results given in Table 4

Table 4

Variable	Coefficient	t –Statistic
Constant (β_0)	16.47	1.20
X_1 (β_1)	.58	3.27**
D_1 (α_1)	-10.14	-1.83*
Adjusted R^2 .20		
F 7.1		

Note, ** p< .05; * p< .10

Results in Table (4) indicate that Approach 2 had no significant effect on the impact of NQCP on quiz scores but that Approach 1 actually diminished that impact. This suggests that Approach 1 may have actually confused students in their attempt to understand the relationship between bond prices and interest rates.

Question (2) was addressed by estimating regression equation (5) involving the two large introductory course sections where

β_0 = mean effect of Approaches 1 and 2 combined

X_1 = NQCP for the two introductory course sections

D_1 = 1 for all 3 Approaches combined and 0 otherwise

Results for this latest estimation are given in Table 5 below.

Table 5

Variable	Coefficient	t –Statistic
Constant (β_0)	24.06	2.62
X_1 (β_1)	.56	4.21**
D1 (α1)	-9.61	-2.76**
Adjusted R^2 .11		
F 13.60		

Note, ** p< .05; * p< .10

As illustrated in Table (5), the mean effects of both combinations on the impact of NQCP on quiz scores were significant. However, that of the combination of Approaches 1 and 2 was positive while that of the combination of all 3 Approaches was negative. In short, results suggest that the combination of all 3 Approaches was inferior to the combination of Approaches 1 and 2. This may be because the addition of the joint markets explanation itself served to diminish student understanding or that diminishing returns set in as the number of Approaches employed in the explanation of the inverse relationship between bond prices and interest rates increased from two to three.

The final question posed above, question (3), was addressed using all four class sections and the estimates of the effectiveness of all of the explanations that were employed during the fall 2009 semester using regression equation (6)

$Y = \beta_0 + \beta_1 X_1 + \alpha_1 D_1 + \alpha_2 D_2 + \alpha_3 D_3 + \varepsilon$ where (6)

β_0 = mean effect of Approaches 1 and 2 and 3 combined

X_1 = NQCP for the two introductory course sections

D_1 = 1 for Approach 1 and 0 otherwise

D_2 = 1 for Approach 2 and 0 otherwise

D_3 = 1 for Approaches 1 and 2 combined and 0 otherwise

Results for the estimation of equation (6) are given in Table 6 below.

Table 6

Variable	Coefficient	t –Statistic
Constant (β_0)	9.03	1.14
X_1 (β_1)	.64	5.42**
D1 (α_1)	-8.40	-1.55
D2 (α_2)	-0.81	-0.15
D3 (α_3)	9.45	2.85**
Adjusted R^2 .13		
F 10.73		

Note, ** p< .05; * p< .10

Results in Table (6) indicate that the combination of Approaches 1, 2 and 3 (the mean effect given by β_0) did not add significantly to the impact of NQCP on quiz scores. Moreover and consistent with the results in Table 4, Approach 1 appears to be inferior to Approach 2. Finally, the results indicate that the mean effect of the combination of Approaches 1 and 2 significantly enhanced the impact of NQCP on quiz scores compared to all of the other Approaches tested during the fall 2009 semester either alone or in combination.

SUMMARY AND CONCLUSION

In this paper we presented three approaches to explaining the inverse relationship between bond prices and interest rates along with results of estimates designed to test the relative effectiveness of selected approaches alone and in combination. The approaches described herein provide a slightly different perspective from that typically offered in popular textbooks on financial markets and can be identified as relying on the following key points:

- **Approach 1: Fixity of (Coupon) Payments**
- **Approach 2: Bond Price as Sum of Fractions with Fixed Numerators**
- **Approach 3: Joint Markets for Bonds and Loanable Funds**

Results of regression estimates in which dummy variables were used to represent the various approaches yielded the following:

(1) In a comparison between Approaches 1 and 2, the former appears to have interfered (through perhaps adding confusion?) with student understanding compared to the latter.

(2) In a comparison between the combination of Approaches 1 and 2 and the combination of Approaches 1, 2 and 3, the former made a significant positive contribution to the impact of NQCP on quiz scores while the latter made a significant negative contribution. In short, student understanding suffered either because of Approach 3 itself or because of some type of diminishing returns from adding a third approach.

(3) In a comparison between Approach 1, Approach 2, the combination of Approaches 1 and 2, and the combination of all three Approaches, results were consistent with the other estimates. In particular, Approach 1 seems to be inferior to Approach 2, and unlike the impact of the combination of all three Approaches, the combination of Approaches 1 and 2 did serve to enhance the impact of students' NQCP on quiz scores.

In short, this paper offers somewhat non-standard approaches to explaining the inverse relationship between bond prices and interest rates and presents results that suggest that particular approaches can indeed make a difference in students' understanding of this key financial markets relationship.

Unfortunately, class schedules place restraints on the opportunity to present various approaches and then to subsequently test their effectiveness. In addition to extending the analysis presented in this paper to a greater number of students, obvious research going forward would involve presentation of Approach 3 by itself along with combinations of approaches other than those reported on in this paper.

Regardless of any specific quantitative results, however, different instructors may find that different approaches offer a better fit for both them and their students. What this paper has done is to provide a variety of atypical approaches from which to experiment and select along with statistical results that suggest that the particular approach(es) used can make a difference in helping students understand the inverse relationship between bond prices and interest rates.

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