Defense Spending and Economic Growth:  
An Endogenous Growth Model

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ABSTRACT

This paper explores the impact of the government’s resources allocation between the defense sector and the non-defense sector on economic growth in an endogenous growth model. We prove that shifting resources from the defense sector to the non-defense public sector will stimulate the steady-state growth rate, confirming the empirical findings of Abu-Bader and Abu-Qarn (2003).  

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INTRODUCTION

The effect of the government’s resources allocation between the defense and the non-defense sectors on economic growth has been an attracted considerable subject of intensive discussion in recent years. In a recent article, Abu-Bader and Abu-Qarn (2003) provide pieces of empirical evidence that is based on a cross-section data of Egypt, Israel and Syria, for the past three decades. They find that the defense burden negatively affects economic growth, and that the civilian government expenditures cause positive economic growth. Many empirical studies also provide the same findings, for example Deger and Smith (1983), Faini et al. (1984), Deger (1986), Mintz and Huang (1990), Ward and Davis (1992), and Lipow and Antinori (1995). In other words, reallocating resources from defense to productive civilian spending may stimulate economic growth. Hence, this paper attempts to set up an endogenous growth model to explain the empirical findings of Abu-Bader and Abu-Qarn.

For the purpose of analysis, it is important to distinguish the composition of government expenditure. We explain the role of the government expenditure from the viewpoint of both the demand-side effect and the supply-side effect. For the demand-side effect, the government consumption expenditure could provide direct utility to households, such as expenditures on national defense, and national parks. Generally speaking, defense spending provides both internal and external security and hence enhances the utility of the representative agent; see Sandler and Hartley (1995). For the supply-side effect, the government infrastructure expenditure could raise the productive capacity of firms, such as expenditures on roads, bridges, airports, highways, dams, communication networks, and other transportation networks; see Futagami et al. (1993). However, the aforementioned government expenditures have different impacts on preferences and technology. Hence, a shift in the composition of government expenditure can have a considerable influence on macroeconomic performance (e.g., economic growth); see Turnovsky and Fisher (1995) and Devarajan et al. (1996).

The character of endogenous growth theories stresses that the long-term rate of growth is an endogenous outcome. Since then, many studies have examined the growth effects of fiscal policy in endogenous growth models. Among endogenous growth literatures, Barro (1990) first regards the current flow of government expenditure as a productive input in private production. After the publication of the Barro’s model, Futagami et al. (1993) modify the Barro model by introducing the stock specification of government expenditure in an endogenous growth model. However, the government infrastructure expenditure could provide the public capital stock and hence enhance private productivity. Therefore, we follow the Futagami et al. viewpoint to specify our analytical framework involving the feature of the government infrastructure expenditure.
This paper sets up an endogenous growth model embodying the features of the demand-side and supply-side effects of the government expenditure, and examines the relationship between defense burden and the economic growth rate. We prove that reallocating resources from the defense sector to the non-defense sector may stimulate economic growth. This conclusion may provide a theoretical explanation for the Abu-Bader and Abu-Qarn (2003) empirical findings. The structure of the paper is as follows. Section 2 presents a simple endogenous growth model. Section 3 concludes the paper.

**MODEL**

We assume a closed economy consisting of a representative agent and a government. The agent produces a single composite commodity which can be consumed, invested, and used as a tax payment. The government allocates the tax revenue to both defense and non-defense sectors.

The preference of the representative agent is defined on its consumption, $c$, and the domestic defense spending, $M$. National defense spending is expected to provide the internal and external security, and thus is included in the utility function. Following Brito (1972), Deger and Sen (1983, 1984), van der Ploeg and Zeeuw (1990), Zou (1995), Chang et al. (1996), Shieh et al. (2002a, 2002b), and Gong and Zou (2003), we specify the instantaneous utility function as

$$U(c, M) = \left(\frac{c^\alpha M^{1-\alpha}}{1-\sigma} - 1\right)^{-\frac{1}{\sigma}}, \quad \sigma > 0, 0 < \alpha < 1,$$

where $1/\sigma$ represents the intertemporal elasticity of substitution. The instantaneous utility function will reduce to $U(c, M) = \alpha \ln c + (1 - \alpha) \ln M$ when $\sigma = 1$.

In line with Barro (1990) and Futagami et al. (1993), we assume that output in the non-defense public sector may have a positive impact on private production due to the fact that it provides infrastructures such as dams, communication networks, transportation networks, and airports. Hence, given constant returns to scale, the production function of the representative agent can be written as

$$Q = kf(G/k), \quad f' > 0, f'' < 0,$$

where the variable $k$ is the private capital stock and the variable $G$ is the non-defense public capital stock. The representative agent’s budget constraint is given by

$$\dot{k} = (1 - \tau)kf(G/k) - c,$$

where the overdot represents the rate of change with respect to time and the variable $\tau$ is a flat-rate income tax.

The representative agent chooses its consumption to maximize the discounted sum of future instantaneous utility, subject to the private capital accumulation condition (3), and with the initial condition given by $k_0$, namely,

$$\max \int_0^\infty \left(\frac{c^\alpha M^{1-\alpha}}{1-\sigma} - 1\right)^{-\frac{1}{\sigma}} e^{-\rho t} \, dt,$$

s.t. $\dot{k} = (1 - \tau)kf(G/k) - c,$

where the parameter $\rho$ is the subjective discount rate, $0 < \rho < 1$. Therefore, the optimal necessary conditions of this optimization problem are given by

$$\frac{\alpha}{c} \left(\frac{c^\alpha M^{1-\alpha}}{1-\sigma}\right)^{1-\sigma} = \lambda,$$  \hspace{1cm} (4a)

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1 - \tau)(1 - \eta)f(G/k),$$  \hspace{1cm} (4b)

together with the agent’s budget constraint (3). In equation (4), the variable $\dot{\lambda}$ is the costate variable which represents the shadow value of the private capital stock measured in utility terms and the variable $\eta \equiv (G/k)f''/f$ represents the output elasticity with respect to the non-defense public capital stock, $0 < \eta < 1$. Equation (4a) states that the marginal utility of consumption and the shadow value of private capital stock must be equal at an optimum. Equation (4b) states the accumulation condition of the shadow value.
Differentiating equation (4a) with respect to time and being substituted into equation (4b), we have

$$\frac{\dot{c}}{c} = \frac{1}{\Delta}[(1 - \tau)(1 - \eta)f(G/k) - \rho],$$  \hspace{1cm} (5)

where $\Delta = 1 - \alpha (1 - \sigma) > 0$. Equation (5) represents the optimal growth of the consumption.

The government collects income tax revenue to finance the government spending $g$, including the national defense spending and government infrastructure expenditure. Let $\theta$ and $1 - \theta$ represent the fraction of government spending devoted to national defense spending and core infrastructure investment, respectively. Therefore, we have

$$g = \theta Q = \theta f(G/k),$$  \hspace{1cm} (6a)
$$M = \theta g = \theta df(G/k),$$  \hspace{1cm} (6b)
$$\dot{G} = (1 - \theta)g = (1 - \theta)\theta f(G/k).$$  \hspace{1cm} (6c)

The variable $\theta$ is the defense burden. Equation (6a) is the government budget constraint denoting the fact that the government always balances its budget at each instant of time.

Following Futagami et al. (1993), Barro and Sala-i-Martin (1995), and Faig (1995), we define $x \equiv G/k$ and $y \equiv c/k$. Therefore, summarizing the equations (3), (5), and (6c), we get the following dynamic equations

$$\dot{x} = \frac{\dot{G}}{G} - \frac{k}{k} = (1 - \theta)\theta f(x)x^{-1} - (1 - \tau)f(x) + y,$$  \hspace{1cm} (7a)
$$\dot{y} = \frac{\dot{c}}{c} - \frac{k}{k} = \frac{1}{\Delta}[(1 - \tau)(1 - \eta)f(x) - \rho - \Delta(1 - \tau)f(x)] + y.$$  \hspace{1cm} (7b)

Equations (7a) and (7b) describe the transitional dynamics of the economy. In characterizing the steady-growth equilibrium of the model, we get that there exists a unique stationary state (i.e., $x^*$, and $y^*$), which satisfies $\dot{x} = \dot{y} = 0$. Appendix states the stability analysis of the dynamic system. However, from equations (7a) and (7b), we have

$$\frac{\partial x^*}{\partial \theta} = \frac{\tau y^*f(x^*)}{\Omega} < 0,$$  \hspace{1cm} (8a)
$$\frac{\partial y^*}{\partial \theta} = \frac{(1 - \tau)y^*f(x^*)[x^*f''(x^*) + \Delta f''(x^*)]}{\Delta \Omega},$$  \hspace{1cm} (8b)

where $\Omega = x^*y^*[-(1 - \theta)(1 - \eta^*)f(x^*)]/(x^*)^2 + (1 - \tau)\Delta x^*f(x^*)] < 0$ (see appendix). Equation (8a) indicates that a rise in the defense burden has a negative impact on the public capital-private capital ratio, $x^*$. Equation (8b) states that the relationship between the defense burden and the consumption-capital ratio $y^*$ is ambiguous.

We then examine the effect of the government’s resources allocation between the defense and the non-defense sectors on the economic growth rate. Let $\gamma^*$ be the steady-state growth rate and note that $(\dot{Q}/Q)^* = (\dot{c}/c)^* = (\dot{k}/k)^* = (\dot{G}/G)^* = \gamma^*$ hold in the steady-growth equilibrium. From equations (3), (8a), and (8b), we get

$$\frac{\partial \gamma^*}{\partial \theta} = (1 - \tau)f'(x^*) \frac{\partial x^*}{\partial \theta} - \frac{\partial y^*}{\partial \theta} = \frac{-\tau(1 - \tau)x^*y^*f(x^*)f''(x^*)}{\Delta \Omega} < 0,$$  \hspace{1cm} (9)

Equation (9) indicates that the relationship between the defense burden and the steady-state growth rate is negative. Namely, shifting resources from the defense sector to the non-defense public sector will stimulate the steady-state growth rate. This result confirms the empirical findings of Abu-Bader and Abu-Qarn (2003). From equation (9), we can indicate the economic intuition behind the story. As stated in the first line in equation (9), a rise in the defense burden will impact the steady-state growth rate through two channels. The first is “the crowding-out effect” whereby a rise in the defense burden will reduce the resources available to the non-defense public sector, and hence reduce private
productivity. Therefore, this channel has a negative impact on the steady-state growth rate. The second is “the resource mobilization effect” whereby a rise in the defense burden will affect the behavior of private consumption. From equation (8b), we get the result that the relationship between the defense burden and the consumption-capital ratio is ambiguous. Therefore, the second channel has an ambiguous impact on the steady-state growth rate. The net effect, as stated in the second line in equation (9), is that a reduction in the defense burden will stimulate the steady-state growth rate.

CONCLUSIONS

In a recent article, Abu-Bader and Abu-Qarn (2003) find the fact that reallocating resources from the defense sector to the non-defense sector may stimulate economic growth. In order to explain the empirical findings of Abu-Bader and Abu-Qarn, our paper sets up an endogenous growth model to explore the relationship between the defense burden and the economic growth rate. We incorporate both the demand-side effect of defense spending proposed by Sandler and Hartley (1995) and the supply-side effect of government infrastructure expenditure proposed by Futagami et al. (1993) into the existing endogenous growth model. We prove that the relationship between the defense burden and the economic growth rate is negative. Namely, we confirm the findings of Abu-Bader and Abu-Qarn.

APPENDIX

We now explore the transitional dynamics. In order to examine the local stability of the dynamic system, we rewrite equations (7a) and (7b) as follows

\[
\dot{x} = [(1-\theta)\overline{f}(x)x^{-1} - (1-\tau)f(x) + y]x, \quad (A.1)
\]

\[
\dot{y} = \frac{1}{\Delta}[(1-\tau)(1-\eta)f(x) - \rho - \Delta(1-\tau)f(x)] + y]y. \quad (A.2)
\]

Linearizing equations (A.1) and (A.2) around the steady-growth equilibrium, we have

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
x - x^* \\
y - y^*
\end{bmatrix},
\]

where

\[
a_{11} = x^*[(1-\theta)\overline{f}(x^*)x^*f'(x^*) - f(x^*)],
\]

\[
a_{12} = x^*.
\]

\[
a_{21} = -(1-\tau)\Delta^{-1}y^*[x^*f''(x^*) + \Delta f'(x^*)],
\]

\[
a_{22} = y^*.
\]

Let the parameter \( \Omega \) represent the determinant of this coefficient matrix. We get

\[
\Omega = a_{11}a_{22} - a_{12}a_{21} = x^*y^*[-(1-\theta)(1-\eta^*)\overline{f}(x^*)/(x^*)^2 + (1-\tau)\Delta^{-1}x^*f''(x^*)] < 0. \]

Hence, one of the eigenvalues of the coefficient matrix is positive and the other is negative. Namely, the steady-growth equilibrium is a saddle point.

REFERENCES


