Importing Fuzzy Measure to Relative Scores of Solar Industry Efficiency

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ABSTRACT

This paper suggests extending scores of technical efficiency as used in, for example, Data Envelopment Analysis (DEA) to efficiency scores defined as so-called fuzzy intervals. Fuzzy scores allow the decision maker to use scores of technical efficiency in combination with other sources of available performance information e.g. expert opinions, key figures etc. In this sense, results of, DEA will be used to control the ‘subjective’ information supplied by expert opinions while the subjective information will be used to supplement the narrow Input-output modeling of the production units used in DEA. So, fuzzy measures become formal images of the evaluation otherwise done implicitly by the decision maker.

Keywords: DEA, fuzzy intervals, Fuzzy scores, Solar Industry.

INTRODUCTION

Data Envelopment Analysis (DEA) is a method capable of evaluating the efficiency of decision making units (DMUs) in utilizing multiple inputs to produce multiple outputs; hence, it is widely applied in evaluating the efficiency of many areas (Kuo et al., 2001). Relative scores of technical efficiency may be a problem for at least three reasons. If, for example, DEA is used to determine the efficiency of a given data set the common reference technology is estimated as the convex envelopment of the data points. Therefore, even technically efficient production units may turn out to be inefficient in a broader (managerial) perspective – the best units in a certain group are not necessarily the best units possible.

Now, merging several sources of information will of course complicate the picture of efficiency. However, we find that the concept of ‘good’ performance is inherently fuzzy and hence might as well be treated so. Our application, on the other hand, where two qualitatively different sources of information are merged by the decision maker seems to fit the idea of a fuzzy score very well and may hence be seen as a natural example of fuzzy intervals. Technical efficiency being measured on a relative scale is not directly comparable to subjective performance judgments which are measured on an absolute scale. However, the membership function of a fuzzy interval may represent one way to specify how these two different types of information are related.

The paper is organized as follows: Section 2, we provided a short motivation, which measured technical efficiency. In Section 3, we define how to express technical efficiency as fuzzy intervals.
Section 4, concerned with a ranking procedure for fuzzy intervals, illustrated by solar industry. Section 5, closed with conclusions.

MEASUREMENT OF TECHNICAL EFFICIENCY

Since 1957, Farrell adopted “The Measurement of Productive Efficiency”, which uses a single input and output to analyze efficiency through mathematical programming in order to find the efficiency frontier curve and to estimate the technical efficiency and price efficiency of the DMU based on the curve. Previous studies have mostly applied the DEA approach owing to assuming that inputs and outputs are crisp values. Therefore, they do not encounter difficulties for the uncertain variables under the DEA model which utilizes a variable return to scale.

Expert evaluations are often used side by side with productivity analysis as e.g. DEA. The following example is part of a confidential project and hence no details concerning the activities will be mentioned. However, DEA was used to determine the efficiency of a sample consisting of 30 similar production units which were modeled by various input-output descriptions. Along with DEA a number of experts were asked to evaluate the performance of the units with respect to various criteria. In Table 1 below a selection of 10 units is shown. A particular DEA model is chosen where efficiency is measured with respect to constant and variable returns to scale technology. Moreover, only expert evaluations on the absolute performance level of each unit are shown. For each unit, subjective efficiency is determined by the interval from the most pessimistic opinion to the most optimistic one within the group of experts. For the sample of 30 production units there are large numbers of DEA efficient units for the chosen model. Average efficiency lies between 0.91 and 0.92. Also, there is a very small difference between the efficiency scores under constant and variable returns to scale. In fact, only one unit becomes efficient changing to a variable return to scale technology.

Concerning the subjective efficiency intervals there seems to be a lower limit of 0.4, but scores as high as 0.9 are assigned. The average scores lie between 0.46 and 0.63. No unit is considered as totally efficient indicating that the values are assigned keeping some sort of ideal production plan in mind. Also the judgments are assigned with relative certainty in the sense that most efficiency intervals are rather narrow. More importantly, however, is the fact that the ranking of units is very different from that of DEA. As an example consider unit A01 in the table above. A01 is assigned efficiency score 1 by DEA (i.e. best performer) but is considered as inefficient by the experts (scores between 0.4 and 0.5). By itself this may be a result of the relativity of DEA but compared to, for example, unit A24 we see that A24 is inefficient according to DEA (scores between 0.91 and 0.93) whereas it is considered more efficient than A1 by the experts (scores between 0.6 and 0.7) - that is, we are left with the opposite ranking. If A01 is compared to A15 we see that they are considered equally efficient by the experts but A15 is in fact the most inefficient unit according to DEA.

<table>
<thead>
<tr>
<th>Unit No.</th>
<th>General DEA</th>
<th>Subjective efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>A11</td>
<td>1</td>
<td>(0.4, 0.7)</td>
</tr>
<tr>
<td>A30</td>
<td>1</td>
<td>(0.5, 0.9)</td>
</tr>
<tr>
<td>A23</td>
<td>1</td>
<td>(0.5, 0.6)</td>
</tr>
<tr>
<td>A01</td>
<td>1</td>
<td>(0.4, 0.6)</td>
</tr>
<tr>
<td>A24</td>
<td>(0.91, 0.93)</td>
<td>(0.6, 0.7)</td>
</tr>
<tr>
<td>A26</td>
<td>(0.86, 1)</td>
<td>(0.5, 0.9)</td>
</tr>
</tbody>
</table>
Ideally then the two sources of performance information ought to be merged such that the ‘objectivity’ of DEA can be used to control the ‘subjectivity’ of the expert statements and vice versa. Such a ‘corrected’ efficiency score may have the form of a fuzzy interval as suggested in the following.

**SCORES OF TECHNICAL EFFICIENCY EXPRESSED AS FUZZY INTERVALS**

Imagine the following scenario: A decision maker (or management) wishes to analyze the performance of a set \( N \) of similar production units where \( i \in N \) is a particular unit. After a preliminary analysis two main sources of information are available: sets of technical efficiency scores, e.g. as a result of DEA or some kind of parametric efficiency analysis, and less formal performance judgments including qualitative aspects, for example, in the form of various expert evaluations.

Although terms like fuzzy sets, membership functions and fuzzy intervals from fuzzy set theory will be used several times in the sequel, no real knowledge of the theory of fuzzy sets is actually required - we shall only consider a few necessary definitions. For an introduction to fuzzy sets the reader is referred to Dubois and Prade (1980) or Kaufmann and Gupta (1991).

As mentioned in Section 1, scores of technical efficiency are crucially linked to a specific input-output model which often is much too limited in dimension to capture the true variation in activity. Hence, it was argued that additional information, for example, in the form of expert judgments ought to supplement the scores of technical efficiency.

Suppose that there are \( H \) DMUs to produce \( M \) outputs by using \( N \) inputs. In Fuzzy DEA, inputs and outputs are characterized by uncertainty, and therefore we use \( \tilde{X}_{jn} \) and \( \tilde{Y}_{jm} \) to represent input \( n \) and output \( m \) of DMU \( j \), respectively. Assume that the inputs \( \tilde{X}_{jn} \) and the outputs \( \tilde{Y}_{jm} \) are approximately known and can be represented by membership functions \( \mu_{\tilde{X}_n} \) and \( \mu_{\tilde{Y}_m} \) of the fuzzy set, respectively.

Note that the crisp value can be represented by degenerated membership functions in which there is only one value in their domain. The Fuzzy DEA model can be written as follows:

\[
\tilde{E}_j = \max \left( \frac{\sum_{m=1}^{M} u_m \tilde{Y}_{mj} - u_0}{\sum_{n=1}^{N} v_n \tilde{X}_{nj}} \right) \left| \frac{\sum_{m=1}^{M} u_m \tilde{Y}_{mh} - u_0}{\sum_{n=1}^{N} v_n \tilde{X}_{nh}} \leq 1 \right| \quad h = 1, \ldots, H
\]

where \( u_m, v_n \geq \varepsilon > 0, \ u_0 \) is free.

Where \( \varepsilon \) is a small non-Archimedean quantity. Kao and Liu (2000) proposed a way to transform the Fuzzy DEA model to the traditional crisp DEA model by applying \( \alpha \)-cut approach. The \( \alpha \)-cut of \( \tilde{X}_{jn} \) and \( \tilde{Y}_{jm} \) are defined as follows:
\[ (X_{jn})_{\alpha} = \{ x_{jn} \in S(\tilde{X}_{jn}) \mid \mu_{\tilde{X}}(x_{jn}) \geq \alpha \} = [(X_{jn})_{\alpha}^{L}, (X_{jn})_{\alpha}^{U}] \]
\[ = \min\{ x_{jn} \in S(\tilde{X}_{jn}) \mid \mu_{\tilde{X}}(x_{jn}) \geq \alpha \}, \max\{ x_{jn} \in S(\tilde{X}_{jn}) \mid \mu_{\tilde{X}}(x_{jn}) \geq \alpha \} \]
\[ (Y_{jm})_{\alpha} = \{ y_{jm} \in S(\tilde{Y}_{jm}) \mid \mu_{\tilde{Y}}(y_{jm}) \geq \alpha \} = [(Y_{jm})_{\alpha}^{L}, (Y_{jm})_{\alpha}^{U}] \]
\[ = \min\{ y_{jm} \in S(\tilde{Y}_{jm}) \mid \mu_{\tilde{Y}}(y_{jm}) \geq \alpha \}, \max\{ y_{jm} \in S(\tilde{Y}_{jm}) \mid \mu_{\tilde{Y}}(y_{jm}) \geq \alpha \} \]  

(2)

Where \( \alpha \in [0, 1] \), and \( S(\tilde{X}_{jn}) \) and \( S(\tilde{Y}_{jm}) \) are the support of \( \tilde{X}_{jn} \) and \( \tilde{Y}_{jm} \), respectively. The intervals indicate the corresponding inputs and outputs ranges at each possibility level \( \alpha \). According to Zadeh’s extension principle (1978), the membership function of efficiency evaluation for DMU \( j \) may be defined as follows:
\[ \mu_{\tilde{E}}(z) = \sup_{x,y} \min\{ \mu_{\tilde{X}}(x_{jn}), \mu_{\tilde{Y}}(y_{jm}), \forall j, m, n \mid z = E_j(x, y) \} \]  

(3)

Where \( E_j(x, y) \) is the efficiency score calculated by the conventional BCC model under a set of \( x \) and \( y \). In order to get the bounds of the intervals for \( \tilde{E}_j(x, y) \) at each possibility level \( \alpha \), Kao and Liu (2000) suggest a pair of mathematical programming, which is function of \( \alpha \), as follows:
\[ (E_j)_{\alpha}^{L} = \max \left( \frac{\sum_{m=1}^{M} u_{m} (Y_{jm})_{\alpha}^{L} - u_{0}}{\sum_{n=1}^{N} v_{n} (X_{jn})_{\alpha}^{U}} \right) \]  

s.t. \[ \frac{\sum_{m=1}^{M} u_{m} (Y_{jm})_{\alpha}^{L} - u_{0}}{\sum_{n=1}^{N} v_{n} (X_{jn})_{\alpha}^{U}} \leq 1 \]

\[ \frac{\sum_{m=1}^{M} u_{m} (Y_{hm})_{\alpha}^{L} - u_{0}}{\sum_{n=1}^{N} v_{n} (X_{hn})_{\alpha}^{U}} \leq 1, \]  

\[ h = 1, \ldots, H, \; h \neq j, \; u_{m}, v_{n} \geq 0, \; u_{0} \text{ is free.} \]  

(4a)

\[ (E_j)_{\alpha}^{U} = \max \left( \frac{\sum_{m=1}^{M} u_{m} (Y_{jm})_{\alpha}^{U} - u_{0}}{\sum_{n=1}^{N} v_{n} (X_{jn})_{\alpha}^{L}} \right) \]  

s.t. \[ \frac{\sum_{m=1}^{M} u_{m} (Y_{jm})_{\alpha}^{U} - u_{0}}{\sum_{n=1}^{N} v_{n} (X_{jn})_{\alpha}^{L}} \leq 1 \]

\[ \frac{\sum_{m=1}^{M} u_{m} (Y_{hm})_{\alpha}^{U} - u_{0}}{\sum_{n=1}^{N} v_{n} (X_{hn})_{\alpha}^{L}} \leq 1, \]  

\[ h = 1, \ldots, H, \; h \neq j, \; u_{m}, v_{n} \geq 0, \; u_{0} \text{ is free.} \]  

(4b)

Model (4a) means that in order to find the minimal relative efficiency of the DMU \( j \) compared with others, the data should be applied from the lowest output value of the DMU \( j \) and the lowest input value of other DMUs, as well as the highest input value of the DMU \( j \) and the highest output value of other DMUs. Similarly, model (4b) indicates that for the maximal relative efficiency of the DMU \( j \) compared with others, the data should be applied from the highest output value of the DMU \( j \) and the highest input value of other DMUs, as well as the lowest input value of the DMU \( j \) and the lowest output value of other DMUs. Furthermore, model (4a) and (4b) are conventional DEA models, capable of translating into linear programming to get the optimal weights. The \( \alpha \)-cut set of efficiency score \( \tilde{E}_j \) may be established as \( (E_j)_{\alpha} = [(E_j)_{\alpha}^{L}, (E_j)_{\alpha}^{U}] \).

Fuzzy DEA may result in a fuzzy efficiency score. Picking the best DMU from numerous fuzzy efficiency scores cannot be solely determined by the fuzzy efficiency score. Therefore, ranking fuzzy
efficiency score becomes the key to find the best DMU. Since the fuzzy efficiency score lies in an interval, we may use \( \alpha \)-cut to rank the fuzzy efficiency score. Different \( \alpha \) value means different range and the level of uncertainty of the efficiency score. The greater the \( \alpha \) value is, the smaller the range of upper and lower bounds is, and the lower the level of uncertainty is. The value \( \alpha = 0.0 \) means the widest range that the efficiency score will emerge, and \( \alpha = 1.0 \) means the efficiency score that is most likely to be. There are many ranking methods for fuzzy numbers. However, most of them need to know the membership functions. We adopt the approach, proposed by Chen and Klein (1997), to rank the fuzzy numbers only based on \( \alpha \)-cut. Let \( k \) be the maximum height of \( \mu_{E_j, j=1,...,H} \).

Suppose that \( k \) is equally divided into \( R \) intervals such that \( \alpha_i = ik/R, \ i = 0,...,R \). Chen and Klein (1997) defined the following index to rank the fuzzy efficiency scores:

\[
I_j = \frac{\sum_{i=0}^{R} ((E_j)_{a_i}^U - c)}{\left( \sum_{i=0}^{R} ((E_j)_{a_i}^U - c) - \sum_{i=0}^{R} ((E_j)_{a_i}^L - d) \right)}, \ R \to \infty
\]  

(5)

where \( c = \min_{i,j} \{ (E_j)_{a_i}^L \} \) and \( d = \max_{i,j} \{ (E_j)_{a_i}^U \} \).

In this sense the parameter \( \alpha \) indicates how much weight the decision maker puts on the (DEA) efficiency result \( E \); relative to that of the subjective scores. If \( \alpha = 1 \) scores in \( E \) are considered at least just as important as scores in \( \mu_{E_j} \); if \( \alpha = 0.5 \) they are considered at least half as important.

Therefore the decision maker ought to choose the same \( \alpha \) value for all units in the analysis, i.e. \( \alpha \) becomes a general statement of the weight of the DEA result relative to the subjective result throughout the analysis.

The increasing and decreasing parts of the membership function which the decision maker uses to combine the kernel and the support can be interpreted as expressing the extent to which the decision maker questions the information supplied by intervals \( \mu_{E_j} \) and \( E \). As mentioned above, this also reflects how relative information is compared to absolute information. An example will help to clarify the issue.

**EMPIRICAL APPLICATION FOR SOLAR INDUSTRY**

Solar cells, also called photovoltaic cells, is a type of photovoltaic component capable of energy conversion which, when exposed under sunlight, will convert the energy of light into electricity. There are a wide variety of solar cells when categorized by the materials they are made of, such as single crystal silicon, polysilicon, amorphous silicon(a-Si), groups III-IV elements (including GaAs, InP, InGaP), groups II-VII elements (including CdTe and CuInSe2), and etc. The upstream materials of solar photovoltaic industrial chain include polysilicon materials, silicon chips, base materials of solar cells (for example, glass, flexible base materials, gases, target materials, slurry, dyes, electrode materials…), and etc; materials for midstream of solar photovoltaic industrial chain include solar cells and photovoltaic modules; for downstream, solar photovoltaic systems and various applications.

The market of global solar photovoltaic cells increased 87% in 2008 than that of 2007. Crystal silicon solar cells were still the mainstream, accounting for 87.5% of the market. Film solar cells, mainly a-Si, CdTe, CIGS, took a proportion of 12.5%. First Solar is rapidly capturing the market of film solar cells with its high efficiency (10.8%) and low cost (below 1 dollar/Wp) and expanding its production capacity to 1 GWp in 2009. Presently the top five producers, ranked by order, are China, Germany, Japan, Taiwan, and the United States. These countries take a proportion of 80% of global
production. With regard to installed capacity, the market growth rate of Spain, which is 348%, is the highest, much higher than expected.

The financial crisis of 2009 has led to difficulty in financing, thus large investment projects are thwarted, and the market is reversed. The prices of polysilicon materials, wafers, solar cells and modules drop substantially as a result of that supply exceeds demand. The prices of downstream systems are also dragged down. Japan and the United States, however, increase investment in green energy related industries in order to stimulate their domestic demands in spite of the market reverse. Thus, the solar photovoltaic industry is expected a bright future even in the recession.

As following description of sola industry, assume that the efficiency of a given sample of production units has been described by a set of fuzzy efficiency scores. There are then various ways to represent preference on that set of fuzzy scores. We makes an example for illustration, those takes manufacturers of solar industry in Taiwan and Chinese as an example, calculates the data collected on the basis of the function to obtain the upper and lower interval values of each $\alpha$ value, and finds the upper and lower bounds of the efficiency score using model (4). We then rank the calculated upper and lower bounds of the efficiency score to find the best performing solar industry.

The research period is the year 2009. The samples are the annual data of 30 solar manufacturers. The sources of the data are from “national-level R&D organization for applied research in industrial technology” and “Market Intelligence & Consulting Institute”. Those are division of Institute for Information Industry. As follow as above, we divided into four major steps:

**Step 1. Determine fuzzy membership function type**

Based on the concept of the intermediation approach and regarding manufacturers as intermediation institutions, the selected outputs are loan, investment, and other income. The inputs are number of employees, total fixed asset value, and deposits (total deposits in solar industry). We analyze the efficiency of the solar industry with a trapezoidal function from the fuzzy set theory. The trigonometric function regresses into a value when $\alpha = 1$, without the fuzzy phenomena. However, it remains with an interval value when $\alpha = 1$ under the trapezoidal function and that is why we adopt it.

**Figure 1: Trapezoidal Membership Function**

**Step 2. Calculate the upper and lower interval values of the $\alpha$ -cut**

This study assumes that the input variables and the other income of the output variables are constant, considering only loan and investment as being fuzzy. Therefore, the leftmost value and the rightmost
value in the trapezoidal function \((a_1, a_5)\) in Figure 1) must be determined first. In terms of the loan variable, this study places the loan variable in the middle \((a_3)\). Adding and subtracting non-performing loans \(\times\) non-performing loan ratio results in \(a_1\) and \(a_5\). Adding and subtracting one half of non-performing loans results in \(a_2\) and \(a_4\). In terms of the investment variable, considering that investment involves profits and losses from two items, including buying and selling stocks and bills and long-term investment of equity shares, this study places the investment variable in the middle \((a_3)\). Adding and subtracting two times the profit or loss from buying and selling stocks and bills and long-term investment of equity shares results in \(a_1\) and \(a_5\). Adding and subtracting the profit or loss from buying and selling stocks and bills and long-term investment of equity shares results in \(a_2\) and \(a_4\). The fuzzy output of \(\alpha\) -cut, to any \(\alpha \in [0,1]\), is expressed as \([(a_1 - a_2)\alpha + a_1, -(a_5 - a_4)\alpha + a_5]\) by the interval method. Insert \(a_1 \sim a_5\) respectively, and match up with 11 \(\alpha\) values, i.e. \(\alpha = 0.0, 0.1, \ldots, 1.0\), to find the upper and lower interval values of loan and investment of all the sample solar industry so as to calculate the fuzzy efficiency score of each manufacturers.

**Step 3.** Calculate the upper and lower bounds of efficiency score

After the upper and lower bounds of loan and investment of the sample manufacturers are derived in step 2, we can make use of the concept from models \((4a)\) and \((4b)\) to find the lower bound of the relative efficiency score of the manufacturers. Table 1 show the possible upper and lower bounds of efficiency scores of each sample company under each \(\alpha\) -cut. The value \(\alpha = 0.0\) means the range that the efficiency score must fall within, and \(\alpha = 1.0\) means that the efficiency score is most likely to be. For example, Company 5 with an efficiency score will be no less than 0.916, nor will it exceed 0.969 under \(\alpha = 0.0\). The greater the \(\alpha\) value is, the smaller the interval will be between the upper and lower bounds of the efficiency score, and this also shows the efficiency score is most likely to be. Table 2 shows that the minimal value is 0.479 and the maximum value is 1; there are 8 companies (No.1, 3, 4, 7, 11, 16, 20 and 30, respectively) with the upper and lower bounds of an efficiency score of 1. Hence, these 8 companies have efficiency score 1 with a crisp value.

### Table 2: Fuzzy Efficiency Scores of solar industry under \(\alpha\) -cut

<table>
<thead>
<tr>
<th>Unit No.</th>
<th>(\alpha = 0.0)</th>
<th>(\alpha = 0.2)</th>
<th>(\alpha = 0.4)</th>
<th>(\alpha = 0.6)</th>
<th>(\alpha = 0.8)</th>
<th>(\alpha = 1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
</tr>
<tr>
<td>2</td>
<td>(0.588,0.803)</td>
<td>(0.596,0.791)</td>
<td>(0.605,0.780)</td>
<td>(0.613,0.769)</td>
<td>(0.622,0.757)</td>
<td>(0.632,0.746)</td>
</tr>
<tr>
<td>3</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
</tr>
<tr>
<td>4</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
</tr>
<tr>
<td>5</td>
<td>(0.916,0.969)</td>
<td>(0.918,0.966)</td>
<td>(0.921,0.964)</td>
<td>(0.924,0.961)</td>
<td>(0.926,0.958)</td>
<td>(0.929,0.956)</td>
</tr>
<tr>
<td>6</td>
<td>(0.932,1.000)</td>
<td>(0.946,1.000)</td>
<td>(0.960,1.000)</td>
<td>(0.974,1.000)</td>
<td>(0.988,1.000)</td>
<td>(1.000,1.000)</td>
</tr>
<tr>
<td>7</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
</tr>
<tr>
<td>8</td>
<td>(0.870,1.000)</td>
<td>(0.880,1.000)</td>
<td>(0.890,1.000)</td>
<td>(0.900,1.000)</td>
<td>(0.911,1.000)</td>
<td>(0.921,1.000)</td>
</tr>
<tr>
<td>9</td>
<td>(0.928,1.000)</td>
<td>(0.938,1.000)</td>
<td>(0.948,1.000)</td>
<td>(0.958,1.000)</td>
<td>(0.968,1.000)</td>
<td>(0.978,1.000)</td>
</tr>
<tr>
<td>10</td>
<td>(0.762,1.000)</td>
<td>(0.775,1.000)</td>
<td>(0.788,1.000)</td>
<td>(0.801,1.000)</td>
<td>(0.815,1.000)</td>
<td>(0.829,0.985)</td>
</tr>
<tr>
<td>11</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
</tr>
<tr>
<td>12</td>
<td>(0.786,0.855)</td>
<td>(0.791,0.854)</td>
<td>(0.797,0.853)</td>
<td>(0.802,0.852)</td>
<td>(0.808,0.851)</td>
<td>(0.814,0.850)</td>
</tr>
<tr>
<td>13</td>
<td>(0.635,0.768)</td>
<td>(0.642,0.762)</td>
<td>(0.649,0.756)</td>
<td>(0.655,0.750)</td>
<td>(0.662,0.744)</td>
<td>(0.669,0.738)</td>
</tr>
<tr>
<td>14</td>
<td>(0.804,0.861)</td>
<td>(0.807,0.858)</td>
<td>(0.810,0.855)</td>
<td>(0.812,0.852)</td>
<td>(0.815,0.849)</td>
<td>(0.818,0.846)</td>
</tr>
<tr>
<td>15</td>
<td>(0.676,0.843)</td>
<td>(0.683,0.834)</td>
<td>(0.691,0.826)</td>
<td>(0.700,0.818)</td>
<td>(0.710,0.810)</td>
<td>(0.719,0.802)</td>
</tr>
<tr>
<td>16</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
<td>(1.000,1.000)</td>
</tr>
</tbody>
</table>
Step 4. Rank fuzzy efficiency scores:

Table 3 shows that the minimal value is 0.414 and the maximum value is 1. There are 8 solar industry with a fuzzy efficiency score of 1 (they are also efficient under non-fuzzy condition); in other words, these companies are efficient under both fuzzy and non-fuzzy conditions. However, there are 5 companies, named company No.6, 9, 7, 19, and 14 with ranking between 15th and 20th, with efficiency score 1 under non-fuzzy condition, while their fuzzy technical efficiencies sit between 0.917 and 0.989. This may indicate that the Fuzzy DEA approach might have a higher ability to discriminate efficient companies than the conventional DEA approach.

### Table 3: Ranking of Fuzzy Efficiency Score of solar companies

<table>
<thead>
<tr>
<th>Rank</th>
<th>Unit No.</th>
<th>Fuzzy Index of Efficiency</th>
<th>Ranking</th>
<th>Unit No.</th>
<th>Fuzzy Index of Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
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**CONCLUSIONS**
In Sections 3 and 4 above it was shown how to represent and rank the performance of production units considered as fuzzy efficiency scores. Compared with previous studies of evaluating solar company efficiency with the DEA approach, the greatest feature of Fuzzy DEA is that it is able to resolve the uncertainty in the production process, which conventional DEA could not handle.

In case the decision maker uses many experts to judge the performance a suitable aggregation procedure must be found. For all practical purposes it is essential that all experts have the same interpretation of the scale of judgment. In view of this it may turn out to be most fruitful to aim at consensus among the experts in the sense that they all have to agree on some performance interval for each unit.

REFERENCES


