The Extreme Behavior of the TED Spread

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ABSTRACT

We empirically analyze the extreme behavior of the TED spread. By applying extreme value theory (EVT), we characterize the tails of distribution of the TED spread and estimate the probability of occurrences of the extreme quantiles in distribution. We intend to answer the question: Is the movement of spread in a magnitude of, for example, say, 600 basis points predictable? How far can the TED spread boost, say, once in 40 years? These answers are important issue as the TED spread is utilized as the anticipation of markets toward future global economic condition. The empirical results show that the distribution of TED spread is fat-tailed and asymmetric. In addition, observed from the estimator of tail index of the TED spread, it is shown that at least the fourth moment of distribution exists. According to the results of formal statistical tests, we find that the distribution of the TED spread resembles ARCH or student’s t class. Finally, the probabilities of occurrence of extreme quantiles of the distribution of the TED spread are also presented. Probability that the daily TED spread would be as high as levels in 1974, over 600 bps, would be exceeded about once every 41.3 years, on average.

INTRODUCTION

The difference between the three month Eurodollar rate and the three month U.S. Treasury bill (T-bill) rate is known as the TED spread. As Treasury bills are risk free whereas Eurodollar deposits are direct obligations of major international banks, Eurodollar deposits are believed to be more risky to pay a higher interest rate than Treasury bills with a common maturity. Thus, the TED spread is supposed to be an indicator of credit risk as it reflects investors’ views of the relative credit quality of the U.S. Treasury and the corporate borrowers at a particular point in time. The TED spreads usually increase during financial crisis or recession. As default risk is considered to be increasing, investors who lose confidence toward the international banking system will switch to U.S. T-bill as a safety investment target. This is known as “flight to quality” that drives up Eurodollar rate, pulls down T-bill rate and hence increases the TED spread.

The TED spread experienced several sudden spikes in the past three decades. The all time high spread level occurred in 1974 when the Bankhaus Herstatt crisis sent the spread soaring to its highest level ever before, over 600 basis points (bps) to around 682 bps. Since the inflation crisis of 1980 and gold’s spike to $850, the TED spread was sent over 500 bps. In 1982, the TED spread spiked to a level of 433 bps in the Third World Debt Crisis. As the stock market dropped 500 points in 1987, the TED soared from 60 to over 250 bps. The recent credit crunch resulting from the U.S. subprime mortgage securities meltdown, boosts up the TED spread to a level around 240bps which has not happened since 1987, though the TED spread has been oscillated around 40bps in the past two years. The extremity of TED spread in market stress reflected the market’s pessimism toward economic prospect and the rising of credit risk. On the other hand, wide spread would have pronounced the contraction in bank lending which would exacerbate a slowdown in global economic growth. In such adverse economic condition, investors who trade the TED spread for hedging or speculation purpose could base on their expectation toward the change of spread. Therefore, to provide the insight of how likely of the extremities would occur, a better understanding of the extreme behavior of the TED spread is essential.
This paper investigates the extreme behavior of the TED spread. On account of the recent turmoil in the U.S. as well as international financial markets, we intend to answer the question: Is the movement of spread in a magnitude of, for example, say, 600 basis points predictable? How far can the TED spread boost, say, once in 40 years? These answers are important issues as the TED spread is utilized as the anticipation of markets toward future global economic condition.

A majority of literature studying TED spread focused on the relationship between Eurodollar rate and Treasury bill rate, some of them examined the lead-lag relationship (Hendershott, 1967; Kwack, 1971; Levin, 1974; Keen and Hache, 1983; Hartman, 1984; Swanson, 1988), the others investigated the cointegration between the two rates (Fung and Isberg, 1992; Fung and Lo, 1995; Tse and Booth, 1995; Booth and Tse, 1995; Shresthat and Welch, 2001; Lee, Shrestha and Welch, 2007). However, none of them concerned about the extreme movement of the TED spread. The current study tempts to fill the gap and directly looks into the tails of distribution of the TED spread to inspect its extreme behavior.

We apply EVT to examine the tail behavior of the distribution of the TED spread. EVT facilitates to directly look into the tails of distribution without assuming the type of distribution. With well-established and sound statistical background, EVT focuses on tails of distribution, describes the limiting distribution of extremes, allows for asymmetric in tails and can be extrapolated beyond the length of the sample. Many studies (Koedijk and Kool, 1994; Danielsson and de Vries, 1997; Longin, 1996, 2005; Bali, 2003; Bali and Neftci, 2003; Cotter, 2005a, 2005b) had found that EVT can effectively capture the tails of distribution of financial returns.

In EVT, tail index is the measure to describe tail fatness of return distribution. It can be estimated parametrically by maximum likelihood (ML) or non-parametric models. However, either model has their pros and cons. The ML estimation of parametric model assumes that the extreme values follow the limiting distribution exactly which is only approximately the case in finite samples (Hols and de Vries, 1991). In addition, the ML estimators are relatively unstable if small sample size is used in estimation process and the convergence toward the asymptotic normal distribution would be slowly. On the contrary, non-parametric technique does not assume any exact limiting law of statistics but only requires the regular varying property of the density function. Thus, non-parametric model is more efficient (Jansen and de Vries, 1991) and this paper employes non-parametric model for tail index estimation. In the non-parametric realm, the Hill estimator proposed by Hill (1975) has been found to perform relatively well on fat-tailed financial data (Kearns and Pagan, 1997). It is also consistent for a wide range of dependent process (Resnick and Starica, 1998). This paper uses Hill estimator for estimating tail index of the TED spread.

Our empirical results indicate that the tail of distribution of the TED spread is fatter than normal. In addition, distribution of the TED spread is asymmetric. As Hill estimator indicates that the distribution has only moments of orders smaller than \( \hat{\alpha} \), the estimated tail index. It is shown that at least kurtosis of the distribution of the TED spread exists. Besides, formal statistical test shows that the distribution of the TED spread resembles ARCH or student’’t class of distribution. Finally, the probabilities of occurrence of extreme quantiles in the right tail of the distribution of the TED spread are also presented. Probability that the daily TED spread would be as high as levels in 1974, over 600 bps, would be exceeded about once every 41.3 years, on average.

The next section briefly introduces the statistical methods used for tail index estimation and threshold choice methodology applied. Then the empirical results are presented and the concluding remark follows.

**METHODOLOGY**

Let \( X_1, X_2, \ldots, X_n \) be a sequence of iid random variables representing positive or negative daily returns from a common unknown fat-tailed CDF \( F(x) \). Define \( M_l = \max(X_1, X_2, \ldots, X_l) \) as the maximum return in a sample of \( l \) (\( l < n \)) periods. Then the class of fat tailed distributions is characterized by the regular variation at infinity

\[
\lim_{t \to \infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha},
\]

where \( \alpha > 0 \) is the tail index. The second order expansion of the tail for large \( x \) is

\[
1 - F(x) \approx ax^{-\alpha}(1 + bx^{-\beta}), \quad \alpha > 0, \quad \beta > 0,
\]

(2)
where \( a, b \) are real numbers, \( \beta > 0 \) is the second-order parameter.

A prominent method for \( \alpha \) estimation proposed by Hill (1975) is the one suitable for modeling heavy-tailed financial time series. Hill estimator directly computes \( \alpha \) in a maximum likelihood sense without parameterizing the tail of distribution. Denote \( X_{(j)} \) as the \( j \)th order statistics with \( X_{(1)} \geq X_{(2)} \geq ... \geq X_{(m)} \geq ... \geq X_{(n)} \). The Hill estimator is given by

\[
\frac{1}{\alpha} = \hat{\xi}(m) = \frac{1}{m} \sum_{j=1}^{m-1} \ln \left( \frac{X_{(j)}}{X_{(m+1)}} \right),
\]

where \( \hat{\xi} \) is the shape parameter and \( m \) is the number of extreme observations or threshold level. Goldie and Smith (1987) showed that \( \hat{\xi} \) is asymptotically normal with mean \( \alpha \) and variance \( \alpha^2/m \).

Given a threshold \( X_{(m+1)} \), the distribution of first \( m \) th order statistics is (c.f. Embrechts, Klüppelberg, and Mikosch., 2003)

\[
\hat{F}(x) = p = 1 - \frac{m}{n} \left( \frac{X_{(m+1)}}{x} \right)^\hat{\alpha} \quad x > X_{(m+1)},
\]

where \( p \) is a given probability level. Thus, the quantile estimator of a given probability \( p \) denoted by \( \hat{x}_p \) is inverted by Eq. (4):

\[
\hat{x}_p = X_{(m+1)} \left( \frac{m}{(1-p) \times n} \right)^{1/\hat{\alpha}}.
\]

**Threshold selection methods**

Before estimating Hill estimator, the threshold level \( m \) should be selected first. Threshold level defines where the distribution of tail started; however, there is bias-variance tradeoff in the selection process. Too low a threshold with more numbers of extreme \( m \) will lead to bias, whereas too high a threshold with few excesses for modeling will lead to high variance. Hall (1990) suggests a subjective statistical method for optimal threshold choice by subsample bootstrapping method.

According to Hall, the optimal threshold choice \( m_0(n) \) is to minimize the asymptotic mean square error as follows

\[
m_0(n) = \arg \min \limits_m \frac{1}{n} \sum_{j=m}^{n} (\hat{\xi}_n(m) - \xi)^2.
\]

Hall suggested to replace unknown \( \xi \) by full sample Hill estimator \( \hat{\xi}_n \). Setting subsample length as \( n_1 = n^{\gamma} \), \( \gamma < 1 \), he proposed subsample bootstrap to calculate MSE at a given threshold level \( m_1 \). By varying threshold level \( m_1 \), the optimal \( \hat{m}^*_1 \) minimizing MSE:

\[
\hat{m}^*_1 = \arg \min \limits_{m_1 \leq m \leq n_1} \frac{1}{n} \sum_{j=m}^{n} (\hat{\xi}_n(m) - \hat{\xi}_n(m_1))^2 \left( \frac{1}{X_{(1)}}...X_n \right),
\]

where \( \hat{\xi}_n^{b}(m_1) \) denotes subsample bootstrapping Hill estimator, \( \hat{\xi}_n \) denotes consistent Hill estimator. Hall set \( \alpha = \beta \) for unknown \( \beta \), then the optimal threshold level is given by

\[
\hat{m}^* = \hat{m}^*_1 \left( \frac{n}{n_1} \right)^{2\beta/(2\beta+\alpha)}.
\]

**EMPirical RESULTS**

**Preliminary data analysis**

The time span of annualized daily spot rates on 3-month Treasury bills and Eurodollars, starting from 1971/1/4 to 2007/12/31, are collected from the online database of Federal Reserve. The 3-month Treasury bills which are originally quoted at discount rate are converted into bond equivalent yields. The TED spread, a total of 9183 observations, is calculated to take the difference between Eurodollar rate and U.S. T-bill yield. The daily plot of the TED spread is
presented in Figure 1. As show in Fig. 1, the TED spread is more volatile and with sharp movements before 1984 and relative stable after 1984 besides some pikes around 1987, 1998, and 2007.

**Figure 1: Plot of daily TED spread**

Table 1 summarizes events that triggered the extreme TED spread. The all time high spread level was in 1974 when the Bankhaus Herstatt crisis sent the spread soaring to its highest level ever before, over 600 bps to around 682 bps. Since the inflation crisis of 1980 and gold's spike to $850, the TED spread was sent over 500 bps. In 1982, the TED spread spiked to around 433 bps in the Third World Debt Crisis. As the stock market dropped 500 points in 1987, the TED soared from 60 to over 250 bps. The recent credit crunch resulting from the U.S. subprime mortgage securities meltdown boosts up the TED spread to a level around 240bps, a level not seen since 1987. In each situation the TED spread affect by the weaken market confidence toward the banking system and soar to a considerable level. As inspected from the historical spikes, we are interested in whether the movement of spread in a magnitude of level as in 1974 or the spike in 2007, after years of remarkably stable TED spread, is predictable? In the following empirical analysis, we will firstly examine the tai-fatness property of distribution of the TED spread by EVT, then the distribution of the TED spread be discerned by estimated tail index, and finally the probability of occurrences of extreme quantiles at a given level will be estimated.

**Table 1: Events of extreme TED spreads**

<table>
<thead>
<tr>
<th>Spread (bps)</th>
<th>Date</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>682.63</td>
<td>1974/7/16</td>
</tr>
<tr>
<td>2</td>
<td>542.48</td>
<td>1980/3/31</td>
</tr>
<tr>
<td>3</td>
<td>432.78</td>
<td>1982/9/23</td>
</tr>
<tr>
<td>4</td>
<td>281.56</td>
<td>1987/10/20</td>
</tr>
<tr>
<td>5</td>
<td>238.67</td>
<td>2007/12/12</td>
</tr>
</tbody>
</table>

Notes, bps is basis points.

The descriptive statistics shown in Table 2 indicate that sample mean of the TED spread is around 100 bps and sample standard deviation is about 93 bps. During the period studied, the TED spread peaked at its historical highest level around 683 bps in 1974 and was down as low as around 3 bps in 2002. The distribution of the TED spread is positively skewed and with excess kurtosis. These properties could also be verify by viewing histogram of the TED spread in Figure 2. It is shown that the distribution has a long right tail implying that there are positive extremities in right tail. The null hypothesis of normal distribution is rejected as Jarque-Bera statistics is significant at 5% level. These evidences support that distribution of the TED spread is non-normal and leptokurtic. The ADF test rejects the null hypothesis of a unit root indicating that the series of the TED spread is stationary.
Figure 2: Histogram of the TED spread

Table 2: Summary statistics of the TED spread

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Mean</td>
<td>100.3</td>
</tr>
<tr>
<td>Sample Median</td>
<td>66.85</td>
</tr>
<tr>
<td>Max</td>
<td>682.63</td>
</tr>
<tr>
<td>Min</td>
<td>2.89</td>
</tr>
<tr>
<td>Sample Std. Dev.</td>
<td>92.85</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.9279</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>7.6576</td>
</tr>
<tr>
<td>J-B</td>
<td>13989.35</td>
</tr>
<tr>
<td>ADF</td>
<td>-5.1909</td>
</tr>
<tr>
<td></td>
<td>[&lt;0.0001]*</td>
</tr>
</tbody>
</table>

Table 3: Tail index estimates

<table>
<thead>
<tr>
<th>threshold level</th>
<th>250</th>
<th>4.7842</th>
<th>-9.2016*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold</td>
<td>3.4171</td>
<td>(0.3026)*</td>
<td>[&lt;0.0001]*</td>
</tr>
</tbody>
</table>

Notes, sample mean, median, max, min and standard deviation are all quoted at basis points (bps). 1 bps is equal to 0.01%. J-B is the Jarque-Bera statistics follow chi-square distribution with 2 degree of freedom, under the null hypothesis of normal distribution. ADF is the ADF unit root tests conducted by regression with intercept and lag term of 5. P-value is reported in brackets. The symbol * denotes significant at the 5% level.

In this section, by applying non-parametric Hill estimator, we explore the tail behavior of the distribution of the TED spread via tail index ($\hat{\alpha}$). The threshold level $m$, the numbers of extremes to include in estimation, is estimated according to Hall (1990) and is corresponded to a threshold of 3.4171 bps in right tail. The estimated results are presented in table 3. The estimated tail index $\hat{\alpha}$ (4.7842) is positive indicating that the distribution of the TED spread has fatter tail than normal distribution. Hill estimator is an indicator of up to which orders of moments exist in the distribution. The value of $\hat{\alpha}$ points out that at least kurtosis of the distribution of TED spread exists. In addition, $\hat{\alpha}$ can be utilized to discern the type of fat-tailed distribution by statistical test under null hypothesis of $H_0: \hat{\alpha} < 2$ against the alternative $H_a: \hat{\alpha} \geq 2$. More specifically, a value of $\hat{\alpha}$ less than 2 is consistent with the stable Paretoian distribution whereas a value of $\hat{\alpha}$ greater than 2 is consistent with the ARCH process or Student’ $t$ distribution. The testing statistics, in the last column of Table 3, exceeding the critical value of -1.645 and rejecting the null hypothesis indicates that the distribution of the TED spread resembles ARCH or Student’ $t$ class of distribution.

Extreme quantiles

An advantage of extreme value theory is that it allows one to extrapolate outside the samples. It also provides the estimated probabilities of the extreme events such as a 40-year level of spread. In this section, we use the estimated tail
index to estimate the probability of extreme quantiles in right tail to provide further investigation on the extreme events. As a measure to describe tail fatness of distribution, the tail index itself is useful to predict the likelihood of an extreme drop, the issue that the financial market is interested in. We estimate the tail excess probability at given extreme spread levels, such as 400, 500, 600, and 700 bps, by Eq. (4) and present the estimated results in Table 4. According to the results, probability that the daily TED spread would be as high as levels in 1974, over 600 bps, within a given year is 2.42%. Stated differently, on average, the daily TED spread will exceed 600 bps once every 1/0.0242 = 41.3 years. A 40-year spread level is around 500 bps and a spread level exceed 700 bps would occur once every 1/0.0234 = 42.7 years.

<table>
<thead>
<tr>
<th>Table 4: Tail excess probability estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>bps</td>
</tr>
<tr>
<td>probability</td>
</tr>
<tr>
<td>T</td>
</tr>
</tbody>
</table>

Note, probability is the probability that the one day drop of the TED spread exceeds a given extreme level within a given year. T is the number of years that a given extreme daily spread level will occur once every T years.

CONCLUDING REMARKS

This paper investigates the extreme behavior of the TED spread. On account of the recent turmoil in the U.S. as well as international financial markets, we intend to answer the question: Is the movement of spread in a magnitude of, for example, say, 600 bps predictable? How far can the TED spread boost, say, once in 40 years? These answers are important issue as the TED spread is utilized as the anticipation of markets toward future global economic condition.

We employ EVT to examine the tail behavior of the distribution of the TED spread. With well-established and sound statistical background, EVT focuses on tails of distributions, describes the limiting distribution of extremes, allows for asymmetry in tails and can be extrapolated beyond the length of the sample. In EVT, tail index is the measure to describe tail fatness of return distribution which can be estimated either parametrically by maximum likelihood (ML) or non-parametric models. As non-parametric model is more efficient, this paper employs non-parametric model, Hill estimator (Hill, 1975), for tail index estimation. Hill estimator has been found to perform relatively well on fat-tailed financial data and is also consistent for a wide range of dependent processes.

Our empirical results indicate that the distribution of the TED spread is fatter-tailed than normal distribution. In addition, as Hill estimator suggests that distribution has only moments of orders smaller than $\alpha$. It is shown that at least kurtosis of the distribution of the TED spread exists. Besides, formal statistical test shows that the distribution of the TED spread resembles ARCH or student’s $t$ class of distributions. Finally, the probabilities of occurrence of extreme quantiles in the right tail of the distribution of the TED spread are also presented. Probability that the daily TED spread would be as high as levels in 1974, over 600 bps, would be exceeded about once every 41.3 years, on average.

REFERENCES