How Taste Dispersion Affects Optimal Prices, Advertising Expenditures, and Profits in a Dynamic Duopoly

Yao- Hsien, Lee, Associate Professor, Department of Finance, Chung Hua University
Chien- Shiun, Chen, Ph. D. Candidate, Institute of Management of Technology, Chung Hua University
Sheu-Chin Kung, Ph. D. Candidate, Institute of Management of Technology, Chung Hua University

ABSTRACT

The purpose of this paper is to introduce the taste dispersion in disutility and to adopt a more flexible taste distribution form for measuring consumer heterogeneity in the framework of predatory advertising. The results show that greater taste dispersion leads to higher prices and advertising expenditures and the number of consumers. We use numerical illustrations to show managerial implications and give some insights into the steady state prices and advertising expenditures in the context of a symmetric feedback information structure.

Keywords: Advertising expenditures, Taste dispersion, Differential game, Feedback Nash Equilibrium.

INTRODUCTION

Much has been done to consider the effects of advertising within the context of dynamic oligopoly in applying differential game models to investigate optimal advertising and pricing decisions. In the areas of optimal pricing and advertising there is now a considerable body of literature; the reader is referred to the survey articles by Sethi (1977), Little (1979), Friedman (1983a), Feichtinger and Jorgensen (1983), Eliashberg and Chatterjee (1985), Dolan et al. (1986), Jorgensen (1986), Feichtinger et al. (1994), Erickson (1995), and Dockner et al. (2000, chapter 11). The above literature has indicated that previous advances in obtaining a useful understanding of oligopolistic dynamic behavior stem from two primary sources: noncooperative (or cooperative) game theory and economic dynamics. However, elements from both areas have been brought together with economics and management science mostly in the dynamic models of noncooperative game. The model-based literature on dynamic predatory advertising is sparse.

A model of oligopoly with predatory advertising may be found in Friedman (1983a,b). In the terminology of Friedman (1983a, pp.144), advertising that increases by one firm causing a decreasing in the sales of all of its rivals and has no effect upon market demand is called “purely predatory”. Seldon and Doroodian (1989) and Seldon et al. (1993) use statistical tests to support the hypothesis that cigarette advertising is purely predatory and there are positive spillovers from advertising in the cigarette industry. In other words, they find evidence that, while cigarette advertising rearranges market share, it can increase the aggregate market for cigarettes as well. Roberts and Samuelson (1988) also find that advertising in the cigarette industry is highly cooperative and it can both increase market size and affect market shares. Gasmi et al. (1992) estimate a low advertising effect on market size and a strong predatory effect on market shares in the soft drink industry. Slade (1995) obtains that advertising is mildly predatory for saltine crackers. Espinosa and Mariel (2001) develop a dynamic model of oligopolistic advertising competition to include predatory advertising and informative advertising. Their results show that for the predatory advertising competition game, expenditures are inefficiently high in a feedback equilibrium and the open-loop solution is more efficient. Piga (1998) analyses a differential game of duopolistic competition with a differentiated product where firms can use advertising and price as competitive tools. Piga assumes a market characterized by the fact that advertising can either be cooperative or predatory because advertising can both increase market size and affect market shares. In contrast to previous works, following the linear approach of Hotelling (1929), the consumers are assumed to be uniformly distributed on the interval [0,1] with a density equal to N consumers per unit length. Hence, the total number of
consumers in the market is \( N \). This implies that the demand side of a market consists of a large number of consumers with identical tastes and income levels. In a spatial monopoly framework, Lambertini (2005) consider a market for horizontally differentiated products where consumers are assumed uniformly distributed with unit density along the unit interval \([0,1]\). As a consequence, the above authors can not address the dependence of the firms’ choices of price and advertising expenditure and corresponding the number of consumers on the taste and income parameters. Therefore, the interest of exploring this dependence to conducting a comparative static analysis showing how changes in the firms’ choices of price and advertising expenditure when taste distribution variations included in the model has been ignored.

This paper explicitly incorporates the consumer’s taste dispersion parameter and adopts the assumption that each consumer is indexed by \( x \in [a, b] \) in the symmetric case with advertising be both predatory and cooperative as in Piga (1998). Our objective is to examine the impact of taste dispersion on the firms’ choices of price and advertising expenditure and the number of consumers in the market. It has been seldom the case to fully include in a dynamic model of advertising and product differentiation, as it is done here, both pricing and advertising strategies. By adopting a dynamic differential game framework as in Piga, we can provide a full analytical characterization of the equilibria and to compare the market outcomes of Piga’s model. It is of interest to note that the occurrence of higher prices, advertising expenditures, and the number of consumers contrasts with findings established in the Piga’s setting, in which the taste dispersion issue has not been addressed. We find that greater taste dispersion can increase advertising expenditure, marketing size and market shares and improve duopolistic profits. We believe the issue should be interesting, both from a theoretical and a managerial point of view.

Although some recent studies of cooperative advertising in a dynamic marketing channel include dynamic effects of advertising (e.g., Chingtagunta and Jain (1992), Jorgensen and Zaccour (1999), Jorgensen et al., (2000), and Jorgensen et al., (2001)), these works did not address the taste dispersion issue. This paper will not provide a detailed state-of-the-art report on all these developments; we confine ourselves to investigate the nature of dynamic predatory advertising and price strategies applying a differential game of duopolistic firms in the presence of variations in the taste distribution.

The rest of the paper is organized as follows. In section 2, we modify the symmetric case when advertising is predatory in the model of Piga by incorporating the taste dispersion to show how the firms’ choices of price and advertising expenditure and the number of consumers vary with it. In section 3, we compare the results obtained in the paper with those of Piga and interpret the results of comparison by conducting a numerical illustration. In section 4, we conclude.

**THE BASIC SETUP**

The theoretical framework adopted in this paper modifies that of Piga (1998, pp. 517) by allowing for the presence of taste dispersion. We assume that the consumers in the market are uniformly distributed on the interval \([a,b]\) with density \( N \) of consumers per unit length. Since each consumer is indexed by \( x \in [a, b] \), where \( b > a \geq 0 \). Let \( \hat{x} \) denote the consumer who is indifferent to whether he or she purchases from either firm 1 or 2. Formally, if \( a < \hat{x} < b \), then \( \hat{x} \) satisfies the following identity: \( v - p_i - k(\hat{x} - a - \gamma A_i) = v - p_j - k(b - \hat{x} - \gamma A_j) \). Hence, 

\[
\hat{x} = \frac{a + b}{2} + \frac{p_j - p_i}{2k} + \frac{\gamma}{2} (A_i - A_j)
\]  

Where \( \gamma > 0 \) is a measure of the effect of predatory advertising. A larger value of \( \gamma \) implies a larger predatory effect for advertising. As usual, (1) is the demand function faced by firm 1. The demand function faced by firm 2 is \( b - \hat{x} \). It is easy to see that firm 1’s market share is \( y_1 = \hat{x} - a / b - a \) and firm 2’s market share is \( y_2 = b - \hat{x} / b - a \). Therefore, the market shares of firm 1 and 2 can be expressed as follows:

\[
y_i = \frac{1}{2} + \frac{p_j - p_i}{2(b - a)k} + \frac{\gamma (A_i - A_j)}{2(b - a)} \quad i, j = 1, 2 \quad i \neq j
\]
A simple calculation shows that \( b - a = 2\sqrt{3}\sigma \), where \( \sigma \) is a measure of taste dispersion, which is also the standard deviation of the taste distribution. Thus, the demands for output of firms 1 and 2 are given by
\[
N_{y_1} = N \times \frac{x - a}{b - a} \quad \text{and} \quad N_{y_2} = N \times \frac{b - x}{b - a}.
\]

Following in Piga (1998), both firms have different cost structures with constant unit of producing the product, \( c_i \). Total production costs, therefore, of firms are then given by \( TC_i = Nc_i y_i \). Because the most interesting case is one in which both firms have positive demand, assume that \( 0 < y_i < 0 \). A constraint that guarantees this is \( 6\sqrt{3}\sigma k > |c_i - c_j| \). Let \( A_i \) represent the advertising outlay for firm \( i \). The advertising cost \( C_i(A_i) \) is assumed convex increasing and taken, for tractability, quadratic as follows:
\[
C_i(A_i) = \mu A_i^2, \quad i = 1, 2, \tag{3}
\]
where \( \mu > 0 \) is a positive constant.

The number of consumers \( N \) evolves according to the dynamics as in Piga’s model:
\[
\dot{N}(t) = \frac{dN(t)}{dt} = \alpha [A_i(t) + A_j(t)] - \lambda N(t), \tag{4}
\]
where \( \alpha > 0 \) reflects the efficiency of the advertising investment and \( \lambda > 0 \) is the constant decay rate of the number of consumers. It can be seen that (3) implies that the firms are assumed to benefit jointly from increased market size with the sum of their advertising expenditures. In other words, the advertising expenditure can be interpreted as a public good. The analogous implication can also been found in Fershtman and Nitzan (1991).

Assuming that both firms are profit maximizers, their optimization problems read as follows:
\[
\max_{A_i(t), p_i(t)} \int_0^\infty e^{-\rho t} N \left( \frac{1}{2} + \frac{p_j(t) - p_i(t)}{4\sqrt{3}\sigma k} + \frac{\gamma(A_i - A_j)}{4\sqrt{3}\sigma} \right) (p_i(t) - c_i) - \mu A_i^2(t) dt \quad i = 1, 2 \tag{5}
\]
where \( \rho > 0 \) is the common discount rate.

By (5) and (4) we have defined a two-player, non-zero sum, infinite-horizon differential game with four control variables \((p_1, p_2, A_1, A_2)\) and one state variable \((N)\).

Here, Nash equilibria in feedback strategies are considered, where the strategies are differentiable state-dependent rules. That is, the optimal solution when each firm takes into account that, at any time, the control variable of the other firm affects the state variable, requiring a revision of the optimal advertising expenditure plans. In contrast to the open-loop Nash equilibrium, in this model, a firm cannot itself in advance to any given advertising expenditure path. The optimal feedback strategy indicates the best response for each value of the state variable, \( N \), at each time in point. We shall from now on eliminate the time argument when no confusion may arise.

**FEEDBACK NASH EQUILIBRIA (FNE)**

In this section, we investigate the feed-back solution of the differential game. We follow the value function approach based on the Hamilton-Jacobi-Bellman equation (see Kamien and Schwartz, 1991, pp. 274). The approach allows the optimal paths for prices and advertising expenditures to be derived. Using this approach the FNE have to be satisfy the following equation:
\[
\rho V_i(N) = \max \left\{ N \left( \frac{1}{2} + \frac{p_j - p_i}{4\sqrt{3}\sigma k} + \frac{\gamma(A_i - A_j)}{4\sqrt{3}\sigma} \right) (p_i - c_i) - \mu A_i^2 + V_j(N) \left[ \alpha(A_i + A_j) - \lambda N \right] \right\}, \quad i, j = 1, 2, i \neq j \tag{6}
\]
where \( V_i(N) \) is the value function to firm \( i (i = 1, 2) \) of the game starting at state \( N \) and the superscript on \( V \) denotes the partial derivative. Because (6) must hold for any admissible state \( N \), the FNE strategies are subgame perfect Nash equilibria. The necessary conditions for the maximization indicated in (6) are:
\[ N \left[ \frac{\gamma}{4\sqrt{3}\sigma} \left( p_i - c_i \right) \right] - 2\mu A_i + \alpha V_i' (N) = 0 \quad i = 1, 2 \] (7)
\[ \frac{N}{2} + \frac{N(p_i - p_j)}{4\sqrt{3}\sigma k} + \frac{N\gamma (A_i - A_j)}{4\sqrt{3}\sigma} - \frac{N}{4\sqrt{3}\sigma k} (p_i - c_i) = 0 \quad i, j = 1, 2; i \neq j \] (8)

Note that (7) and (8) are also sufficient conditions for the FNE strategies because the right-hand side of (4) is concave for P and A. From (7) and (8) it is evident that
\[ P_i = c_i + 2\sqrt{3}\sigma k \] (9)
\[ A_i = \frac{N\gamma k}{4\mu} + \frac{\alpha V_i' (N)}{2\mu} \] (10)

Substituting (9) and (10) into (4) defines a system of partial differential equations (i=1,2):
\[ \rho V_i' (N) = \sqrt{3}\sigma k N - \mu A_i^2 + 2\alpha A_i V_i' (N) - \lambda V_i' (N) N \] (11)

The structure of the game and the form of system (11) leads the following function being proposed as a solution for (11):
\[ V_i (N) = \ell_1 N^2 + \ell_2 N + \ell_3 \quad i = 1, 2, \] (12)

Where \( \ell_1, \ell_2, \) and \( \ell_3 \) are unknown parameters.

Substituting (12) and its partial derivative \( V_i'^N \) into (11) yield the following system of three nonlinear algebraic equations that must hold for every N. This implies that parameters \( \ell_1, \ell_2, \) and \( \ell_3 \) have to satisfy the following equations:
\[ \frac{\gamma^2 k^2}{16} + \left( \frac{jk\alpha - 6}{2} \right) \ell_1 + 3\alpha^2 \ell_1^2 = 0 \] (13)
\[ \sqrt{3}\sigma k + \left( \frac{jk\alpha - 8}{4} \right) \ell_2 + 3\alpha^2 \ell_1 \ell_2 = 0 \] (14)
\[ \ell_3 - \frac{3}{4} \alpha^2 \ell_2^2 = 0 \] (15)

Notice that with these three equations we are able to derive the FNE for the firm’s prices and advertising expenditures. However, they may exist but need not be unique in general. To guarantee the existence of equilibria, sufficient conditions are illustrated in the proposition presented below be the symmetric case: \( c_1 = c_2 = c \) and closed-form equilibrium feedback strategies are obtained for both firms. The symmetry of the solution imposed in the proposition is reasonable, as the state variable N is the same for firms i and j and firms have identical information and parameter values. For simplicity, only the results for the case, \( A_i = A_j \), are presented here.

Proposition 1. (1) The following strategies constitute a symmetric feedback Nash equilibrium in the case of \( \gamma > 0 \):
\[ A_i^s (N) = \alpha l_i N + \frac{jk}{4} N + \frac{ad_j}{2}, \quad \text{where} \]
\[ \ell_1 = \frac{12 - 2\alpha jk - \sqrt{(2\alpha jk - 12)^2 + 48\alpha^2 \gamma^2 k^2}}{24\alpha^2} \] (17)
\[ \ell_2 = \frac{4\sqrt{3}\sigma k}{8 - \alpha jk - 12\alpha^2 l_1} \] (18)

(2) The resulting stationary equilibrium for the number of consumers, \( N^s, \) the advertising expenditure, \( A_i^s \) and price, \( p_i^s, \) is:
\[ N^s = \frac{2\alpha^2 l_2}{2 - 4\alpha^2 l_1 - \alpha jk} \] (19)
\[ A^s_i = \frac{2c d_i}{2 - 4\alpha^2 l_1 - \alpha \gamma k} \]  
\[ p^s = c + 2\sqrt{3}\sigma k \]  

(20) (21)

(3) The above equilibrium is globally asymptotically stable. When 
\[ N(0) \neq N_0 \], the time path of equilibrium number of consumers is 
\[ N(t) = (N_0 - N^s) e^{-\Delta t} + N^s \], where \[ \Delta = 1 - 2\alpha^2 l_1 - \frac{\alpha \gamma k}{2} > 0 \].

One important aspect of Proposition 1 is that the higher in the taste dispersion the higher are the stationary price, advertising expenditure, and the number of consumers. This is because a large dispersion can give each firm an opportunity to use consumer differences in tastes by fitting to a distinctly market segmentation. Consequently, it could be expected that the higher is this taste dispersion, the higher is the levels of price and advertising expenditure needed to get higher profit. This is in sharp contrast to the model of predatory advertising by Piga (1998, Proposition 4.1, pp. 518) who assumes that the consumers are uniform distributed on the unit interval, so that the parameter of the taste dispersion is implicitly assumed as \( \sigma = 0.2887 \). Under this assumption, the taste dispersion can reinforce the firms’ monopoly power and grant exclusive territory has been ignored. In addition, it is noteworthy that the taste dispersion can lower the severity of the free-riding problem observed in the above result, this implication has not been found in Fershtman and Nitzan (1991) and Piga (1998).

It is now interesting to study the effects of an increase in taste dispersion on the stationary number of consumers, \( N^s \), advertising expenditure, \( A^s_i \), and price \( p^s \). It is easy to obtain the following results:

Proposition 2. The stationary number of consumers, advertising expenditure, and price vary with the taste dispersion parameter as follows:

\[ \frac{\partial N^s}{\partial \sigma} > 0 \]  
\[ \frac{\partial A^s_i}{\partial \sigma} > 0 \]  
\[ \frac{\partial p^s}{\partial \sigma} > 0 \]  

Moreover, it can be shown that
\[ \pi^s = \sqrt{3}\sigma k N - \frac{N^2}{4\alpha^2} \]

Equation (25) clearly indicates that the larger taste dispersion, the larger the profit of the firms.

In next section, we perform a numerical analysis to illustrate the effect of taste dispersion.

NUMERICAL ILLUSTRATION

We now make use of the results of Proposition 1 to conduct a numerical analysis. In order to keep the number of illustrations comparable with the figures in Piga (1998), we fix once and for all the value of the following parameters:

\[ k = \lambda = \mu = \alpha = 1, \rho = 0.1 \]

With these parameters, the following figures illustrate the stationary trajectories of \( N^s \), \( A^s_i \), and \( \pi^s \).
Figure 1. The stationary number of consumers

Figure 2. The stationary advertising expenditure of firm i

Figure 3. The stationary profit of firm i
The taste dispersion parameter $\sigma = 0.29$ in Figures 1-3 represents the case considered in Piga(1998, Figures 3 and 4). The stationary equilibria seen in Figures 1-3 confirm the following results.

**Result 1.** Price competition is more relaxed and advertising expenditures are higher for both firms when the taste dispersion of consumers is larger.

**Result 2.** As may be expected, an increase in the taste dispersion leads to a larger number of consumers. As a result, both firms’ advertising expenditures are less affected by their nature of public good in the presence of taste dispersion.

**Result 3.** An increase in the taste dispersion, modeling the fact that consumers are easier to segment by their taste differences, leads to higher profits for both firms.

**Result 4.** The stationary values of price, advertising expenditures, and the number of consumers in Piga(1998) are lower than those of our as the taste dispersion $\sigma > 0.29$.

Results 1-3 are intuitively appealing. This is because increasing in the taste dispersion gives both firms more opportunities to make use of their monopoly power on a captive market segment. Therefore, with the higher taste dispersion, both firms choose to increase prices and advertising expenditures. The higher number of consumers (market size) and profits follow.

**CONCLUDING REMARKS**

We have investigated the sensitivity of variations in the taste dispersion to advertising dynamics and pricing strategies in a dynamic duopoly. We have modified the model of Piga (1998) to consider the impacts of taste dispersion on the firms’ prices and advertising expenditures and the number of consumers in the framework of symmetric case with predatory advertising. In this respect, we have introduced the concept of taste dispersion in a differential game with allowing for the predatory content of advertising. Our main results have several potentially managerial implications. First, the taste dispersion of consumers influences the prices and advertising expenditures of firms and the market size. Second, price competition is less intense under higher values of taste dispersion. Finally, the existence of taste dispersion can reduce the extent of the public good nature in advertising expenditures.

This present model can be extended to examine nonlinear strategies. As mentioned in Tsutsui and Mino (1990), they show the solution obtained by the conventional method may not be unique and suggest that an alternative method for obtaining multiple solutions in games with one state variable. Another interesting problem for further study is to consider the uncertainty of market size in determining advertising expenditures of firms.

**REFERENCES**


Lambertini, Luca (2005), Advertising in a Dynamic Spatial Monopoly, European Journal of operational Research, 166, 547-556.