The Optimal Contract of Online Advertisement: An Agency Perspective

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ABSTRACT

This paper uses a multi-task principal-agent model to describe the strategic interdependence between advertisers and Web publishers. Under the assumption that a powerful advertiser purposes to contract with a Web publisher who care for this business, this paper (i) investigates the optimal online branding advertising pricing for advertiser, (ii) characterizes the value of providing additional performance measure, and (iii) examines the economic impact of risk and exogenous factors on the optimal incentive weight. The results suggest that CPM is incomplete and imperfect representation of Web publishers’ performance. Besides, the payment based on the impressions number and the clicks number acts as substitutes to induce optimal level of effort of Web publisher.

Keywords: principal-agent model, contract theory, informational asymmetries, online advertising

INTRODUCTION

Internet has been the fastest-growing medium in the last few years. As consumers move online, so do advertisers. According to the study of Song (2001), online advertising is an effective branding vehicle. Online advertising can boost offline sales. Technological innovations also have rendered Internet an attractive medium for advertisers. Such interactive and personalization technologies have made the Internet an effective and accountable medium with unlimited creativity (Shen, 2002). As internet continues to take an ever larger share of the marketing budget, the question of how to evaluate, measure and price Web advertising becomes crucial (Shen, 2002; Dreze and Zufryden, 1998; Hoffman and Novak, 1997).

Despite the Internet's phenomenal growth, measurement and pricing practices on the Web are far from being standardized. Given that the Internet enables advertisers to track responses to online advertising, some propose that advertisers should pay for their Internet advertising on the basis of responses or performances (Zhao and Nagurney, 2005; Ephron, 1997; Parsons, 1997). Others argue that such pricing and measurement methods would dismiss online advertising's brand-building value and force Web publishers to assume accountability for the creativity and effectiveness of messages. For the role of the media has traditionally been to offer access to audience, not to share in the responsibility for the quality of the advertisement itself (Zeff and Aronson, 1999; Parsons, 1997). Part of the difficulty of developing universally acceptable standards is that the Web is a highly fragmented medium, with millions of sites and Web pages that accept advertising. The lack of standardized measures has prompted online publishers and interactive advertising networks to come up with many homegrown measures (Shen, 2002).

In the past, researchers have identified several Web pricing and measurement models, such as exposure-based model, interaction-based model and outcome-based model (Ephron, 1997; Shen, 2002; Mangani, 2004; Zhao and Nagurney, 2005; Feng et al., 2007). Exposure-based model is measured as counts of users landing on the source site page carrying the banner ad and includes flat fee and cost per thousand (CPM). It assumes that advertisers pay for impressions or opportunities to see, much like they pay for advertising on television and other media. Interaction-based model is measured as counts of users clicking on the banner and downloading the advertiser's home page. Outcome-based model is measured as counts of users inquiring or purchasing.

In fact, pricing Internet advertising on the basis of responses or performance is fast growing. According to the report of Interactive Advertising Bureau (IAB), 47 percent of 2006 full internet advertising revenues were priced on a interaction basis, 48 percent were priced on a CPM or impression basis and 5 percent were priced on a hybrid basis (PricewaterhouseCoopers LLP, 2007).
As mentioned above, there are many empirical models about online advertising pricing. However, the question of how the strategic interdependence between advertisers and Web publishers affects the optimal price of online advertising is of great interest. As such, the purpose of this study is to set up a theoretical model to examine the optimal online advertising pricing, which incorporates the strategic interdependence of advertisers and Web publishers.

According to the study of Buchwalter and Martin (2003), in U.S., the online ad prices of bulk purchasing, long-term contracts and cost-per-action deals are negotiated which are different from non-negotiated rate card value. As stated before, the Web is a highly fragmented medium. Due to the high number of websites offering advertising space, the web publishers do not have enough market power to affect the price of web advertising (Mangani, 2004). We are particularly interested in the case that how one Web publisher faces a powerful advertiser. Consider one Website offers free services for its virtual community. On the other end of the spectrum there is a powerful advertiser, for example, a big business, syndicate, or a member ofAdvertisers' Association- such as, World Federation of Advertisers or International Advertising Association. The united Advertisers' Association spreads out powerful influence on mass medium. The advertiser can sign a bulk and long-term online advertising contract with the Web publisher for branding. It would be a major revenue for Web publisher if the contract is realized. At the same time, advertiser has many choices of medium besides internet. According to the report of IAB, Internet advertising revenues accounted for merely 5.9 percent of total U.S. ad spending in 2006 (PricewaterhouseCoopers LLP, 2007). In this case, the advertiser will not advertise in a Website unless the features of virtual community are close to his target consumers and the price of advertising is reasonable to ensure the benefit of advertising. The problem facing advertiser is how to measure online branding advertising and what’s the optimal price of online branding advertising for the advertiser.

We classify the Web publisher’s efforts as either mass marketing or target marketing. The mass marketing effort yields “spread virtual community” while the target marketing effort yields the virtual community aiming at advertiser’s target consumers. Web site’s mass marketing effort drives traffic. For example, social network sites (MySpace and Facebook etc.) devote to creating and improving their services to attract visitors. Facebook has about 49 million active members in 2007. Therefore, Facebook becomes an attractive advertising vehicle (Perez, 2007). Web publisher’s target marketing efforts offer advertisers efficient ways to reach the right customers. For example, Facebook designs strategies to help advertisers attract the targeted traffic and display advertisers’ Flyers to target campuses based on age, gender, and network (Facebook, 2007).

There are many reasons to believe that market for Web advertising is characterized by a number of informational asymmetries. In addition to enlarging the number of users landing on the Website and viewing the advertising, the advertiser hopes to effectively target his audience and lift brand awareness. Although, on the Internet, the impressions number (the number of users landing on the source site page carrying the advertising, IN) and the clicks number (the number of users clicking on the advertising with intention to visiting the advertiser's websites, CN) are commonly reported. The statistics or information about Web site that advertisers could find is published for promoting ad space by Web publishers. Both the mass marketing effort level and target marketing effort level of Web publisher are not directly observable by the advertiser. Hence, there is incentive for Web publisher to hide or to boast his effort levels so as to win over the contract of online advertising. Namely, there is a problem so-called moral hazard with the contract of online branding advertising.

Due to the problems of informational asymmetries and moral hazard with the contract of online branding advertising, this paper uses a multi-task principal-agent model to describe the strategic interdependence between advertisers and Web publishers. It uses multiple performance measures to deal with the problem of goal congruence between Web publishers and advertisers. That is, we consider a performance-based pricing scheme in which both the IN (similar to exposure-based model), and the CN (similar to interaction-based model), are used as performance measures. Besides, we focus on the online brand advertising and exclude online order and purchases, as such, ignore the outcome-based model in this paper. Our approach is similar to the multi-task model examined by Holmstrom and Milgrom (1991). Our analysis differs from theirs in that we establish a specific framework to capture the features of Web market specifically: using linear correlation to describe the relationship between the performance measures. Under the assumption that a powerful advertiser purposes to contract with a Web publisher who care for this business, this paper (i)investigates the optimal online branding advertising pricing for advertiser, (ii) characterizes the value of
providing additional performance measure, and (iii) examines the economic impact of risk and exogenous factors on the optimal incentive weight.

We describe the two-stage game as following moves: The advertiser moves first, choosing advertising fee to motivate Web publisher’s efforts in order to maximize net advertising benefit. In the second stage, the Web publisher evaluates this contract and then decides whether to accept and, if so, at what level of efforts. The advertiser rewards the Web publisher according to the observable variables: the IN and the CN.

In the remainder of this paper, Section 2 sets out a basic model of online branding advertising pricing. The economic impacts of variations in performance measures are examined in Section 3. Concluding remarks are provided in Section 4.

**BASIC MODEL**

For ease of exposition, assume that a unique advertiser, the principal, wants to sign a lucrative long-term branding advertising contract with a Web publisher, the agent. But it is too costly for the advertiser to observe the mass and target marketing effort level of the Web publisher directly. Assume that the Web publisher is risk and effort averse: it implies that the Web publisher incurs a cost c to supply efforts. A large firm engages in many different activities with diverse risks, and as such does not worry about the risks associated with any single activity (McMillan 1992). Therefore, the advertiser can diversify risk and is assumed to be risk neutral.

The Web publisher’s preferences are represented by a negative exponential utility function $U(w) = 1 - e^{-w}$, where $r$ is the Web publisher’s absolute risk aversion and $w$ is his net compensation $w = w_1 + w_2 - c$: his compensation minus cost. $w_1$ is the fee advertiser pays for impressions or opportunities to view, much like he pays for advertising on television and in other media. $w_2$ is the fee advertiser pays for users to interact with or click the advertising.

For simplicity and without loss of generality, the advertising payment is assumed to be a linear function of the performance measures:

$$\begin{align*}
\tilde{w}_1 &= \beta_1 \tilde{n}_1, \quad (1) \\
\tilde{w}_2 &= \beta_2 \tilde{n}_2, \quad (2)
\end{align*}$$

where $\sim$ represents random variable, $n_1$ is the impressions number or IN, $n_2$ is the clicks number or CN. We can view $\beta_1$ as the price paid per unit of performance measure IN while $\beta_2$ as the price paid per unit of performance measure CN. That is to say, $\beta_1$ is the incentive based on IN, while $\beta_2$ is the incentive based on CN.

IN is affected by various exogenous factors captured by variable $q$ directly observable by both the advertiser and the Web publisher—for example, the popularity of Internet. It is also affected by the Web publisher’s mass marketing effort, $b$. A higher mass marketing effort would attract more users landing on the source site page carrying the online branding advertising and viewing the advertising. Similarly, CN is affected by various exogenous factors captured by variable $p$, which is observable by both the advertiser and the Web publisher—for example, the advertiser’s reputation. It is also affected by the IN, and the Web publisher’s target marketing effort, $a$. The Web publisher’s target marketing efforts offer advertisers efficient ways to reach the right customers who become more interested in the advertiser’s advertising. As the publisher’s target marketing effort expands, the number of users clicking on the advertising increases as well. Besides, a fixed portion of consumers landing on the source site page carrying the online is bound to click on the advertising regardless of the level of the Web publisher’s target marketing effort. We denote the number of this kind of consumers as BC. Let $\alpha$ be the ratio of the BC to IN. The logarithm of $b$ and $a$ are used to capture the effect of efforts in increasing performance measures at decreasing rates. The relation between the performance measures and Web publisher’s effort levels are assumed to take the form

$$\begin{align*}
\tilde{n}_1 &= q \ln b + \tilde{\epsilon}_1, \\
\tilde{n}_2 &= a \tilde{n}_1 + p \ln a + \tilde{\epsilon}_2, 
\end{align*}$$

where $\tilde{\epsilon}_1 \sim N(0, \sigma^2_1)$ and $\tilde{\epsilon}_2 \sim N(0, \sigma^2_2)$ are random uncontrollable components of the performance measures. The variances of the uncontrollable components are assumed to be independent. Performance measures depend on Web publisher’s efforts as well as random factors.
To simplify the mathematical analysis, let $\Theta = \ln b$ and $\Phi = \ln a$. We assume $c(\Theta, \Phi)$ is separable and quadratic:

$$c(\Theta, \Phi) = \frac{\Theta^2}{2} + \frac{\Phi^2}{2}. \quad (5)$$

Using backward induction to solve the game, we start with Web publisher’s choosing. With normally distributed uncertainty and negative exponential utility, the Web publisher’s problem can be expressed as maximizing the certainty equivalent (CE) of the expected utility (Holmstrom and Milgrom, 1987). In this scenario, the CE is equal to the mean value of net compensation less the risk premium, or:

$$CE = \beta \bar{\eta} + \beta \bar{\eta} - c(\Theta, \Phi) - \frac{\sigma^2}{2}\left(\beta^2 + (\alpha + \beta)\sigma^2 + \beta^2 \sigma^2\right) \quad (6)$$

Substituting equations (1), (2), (3), (4), and (5) into (6) and differentiating with respect to mass and target marketing efforts yields the Web publisher’s optimal efforts choices, or:

$$\Theta^* = (\beta + \alpha \beta)\eta, \quad (7)$$

$$\Phi^* = \beta \eta. \quad (8)$$

The Web publisher’s optimal efforts choice implies that $\beta_1$ and $\beta_2$ act as substitutes which induce mass marketing effort of Web publisher. Besides, the payment based on IN would induce only the Web publisher’s mass marketing effort, but not his target marketing effort.

To ensure that the Web publisher accepts the incentive contract, the advertiser is constrained to meet Web publisher’s reservation utility (normalized to zero) in each level of Web publisher’s efforts. Note that constraints can be expressed as:

$$E(w_t) = \frac{\Theta^2}{2} + r\beta^2 \sigma^2, \quad (9)$$

$$E(w_t) = \frac{\Phi^2}{2} + r\beta_2 S, \quad (10)$$

where $S = \alpha^2 \sigma^2 + \sigma^2$.

Dynamic Logic (2000) has found that online banner advertising raises brand awareness by 6% on average. It demonstrates that online advertising has value even for the banners who do not click-on. We further assume that there exists a linear relation between the expected advertising benefit, $V$, and the expected levels of the performance measures or

$$V = v(n_t) + u(n_t) = k\bar{\eta} + h\bar{\eta}, \quad (11)$$

where $v$ and $k$ represent the expected advertising benefit (the lift in brand awareness) and marginal advertising benefit of viewing the advertising by consumers landing on the Website. $u$ and $h$ represent the expected advertising benefit and marginal advertising benefit of interacting with the online branding advertising by consumers clicking on the advertising. The advertiser chooses incentive weight to maximize net advertising benefit subject to meeting the Web publisher’s rationality constraints (IR) and incentive compatibility constraint (IC). In this light, the advertiser’s maximization problem can be expressed as:

$$\max_{\theta, \phi} \Pi = \mathbb{E}[v(n_t) + u(n_t) - w_t],$$

s.t. $E\left(1 - e^{-\frac{v(n_t) + u(n_t)}{T}}\right) \geq 0$, \hspace{1cm} (IR)

$E\left(1 - e^{-\frac{v(n_t) + u(n_t)}{T}}\right) \geq 0$, \hspace{1cm} (IR)

$\Theta, \Phi \in \arg\max_{\theta, \phi} \mathbb{E}\left[1 - e^{-\frac{v(n_t) + u(n_t)}{T}}\right]$, \hspace{1cm} (IC)

where $\Pi$ is advertiser’s the net advertising benefit. By substituting optimal efforts and appropriate expressions into the advertiser’s objective function, differentiating with respect to $\beta_1$ and $\beta_2$, and invoking simultaneous solution to the equations, the optimal incentive weights on the performance measures can be obtained as follows:
\[ \beta_i^* = \frac{q^i(rKS + kp^i)}{H}, \]
\[ \beta_1^* = \frac{r\sigma^2(AKq^i + hp^i) + hp^2q^i}{H}, \]
where \( K = k + ah, \ H = \left(q^i + r\sigma^2\right)\left(\alpha^2q^i + p^2 + rS\right) - \alpha^2q^i = q^i\left(p^i + rS\right) + r\sigma^2\left(\alpha^2q^i + p^2 + rS\right) > 0, \ S = \alpha^2\sigma_i^2 + \sigma_z^2. \]

In addition to \( \beta_i^* > 0 \), we can find out that \( \beta_1^* > 0 \) (equation (12) and (13)), indicating that using CN as the additional measure of the performance-based pricing scheme would improve the advertiser’s net advertising benefit. Notice that the payment based only on IN could induce only Web publisher’s mass marketing effort, but not his target marketing effort (see Appendix A). Hence we state the following proposition.

Proposition 1.

Using the CN as an additional measure of the performance-based pricing scheme would improve advertiser’s net advertising benefit than using only the IN as the only performance measure.

RISK AND EXOGENOUS IMPACTS

The performance-based pricing scheme imposes risk on Web publishers. The performance measures, IN and CN, are usually not completely under Web publishers’ control. They depend not only on Web publisher’s efforts, b and a (as well as \( \Theta \) and \( \Phi \)), but also on random uncontrollable components, \( \varepsilon_i \) and \( \tilde{e}_i \) (equation (3) and (4)). While a high fee has the advantage of giving strong performance incentive, it has the disadvantage of imposing risk on the Web publisher. Therefore, incentive payment schemes push the risk of unforeseen eventualities onto the Web publisher, the agent.

Differentiating equations (12) and (13) with respect to \( \sigma_i^2 \), we obtain the following results

\[ \frac{d\beta_i^*}{d\sigma_i^2} = -\alpha X - rB - \alpha^2 \sigma_i^2 Z, \]
\[ \frac{d\beta_1^*}{d\sigma_i^2} = X - \alpha^2 \sigma_i^2 \left(Y + Z\right), \]
where \( X = \frac{\alpha q^i \left(\sigma_i^2 K + p^i(k - ah)\right)}{H^2}, \ Y = \frac{\left(\alpha \sigma_i^2 Kq^i + hp^2(r\sigma_i^2 + q^i)\right)}{H^2} > 0, \ B = \frac{\alpha q^i \left(\sigma_i^2 K + p^2 + 2rS\right)}{H^2} > 0, \ Z = \frac{hp^2q^i}{H^2} > 0. \]

The comparative static results of the noise of performance measure IN (\( \sigma_i^2 \)) are summarized in the following proposition and corollaries.

Proposition 2.

As the noise of performance measure IN increases (decreases), the optimal incentive weight on IN and CN may either increase or decrease.

As \( \sigma_i^2 \) increases, the weight on IN and CN depends on the following two effects. First, both uncertainty of IN and CN increase. As \( \sigma_i^2 \) increases, the uncertainty of IN increases (equation (3)) and it gives rise to the increased uncertainty of CN by the BC (equation (4)). Therefore, advertiser must pay more risk premium to the Web publisher to satisfy the Web publisher’s reservation utility (equations (9) and (10)). This leads to an increase in the agency cost. Therefore, gives the advertiser an incentive to decrease weights on IN and CN in order to reduce the agency cost. Second, it also depends on direction of substitution effect. If either variance is relatively high, the principal will place greater emphasis on the substitute performance measures because emphasizing the substitute will mean that the advertiser can satisfy the reservation utility and pay a relatively lower risk premium. Combining the two effects above, as a result, an increase in the \( \sigma_i^2 \), may either increase or decrease the weight on IN and CN. The converse also holds true.
Corollary 2.1.
If the advertising benefit of viewing the advertising per unit IN is larger than that of interacting with the advertising per unit IN, an increase in the noise of performance measure IN (σ₁) would cause the advertiser to place lower weight on the IN.

Proof. If \( k > ah \), then \( X > 0 \) and \( \frac{dB}{d\sigma^2} < 0 \).

As \( \sigma^2 \) increases, other things being equal, the uncertainty of IN is relatively higher than that of CN. If the advertising benefit (the effect of lift in brand awareness) of viewing the advertising per unit IN is larger than that of interacting with the advertising per unit IN, \( k > ah \), then the substitution effect of IN is negative while that of CN is positive, or \( X > 0 \). This is due to that the disadvantage (loss) of advertising benefit resulting from the relative increase in IN’s uncertainty is greater than the advantage (gain) of advertising benefit resulting from the relatively decrease in CN’s uncertainty. Consequently, the advertiser would increase the incentive weight on CN to substitute for IN.

Corollary 2.2.
If the substitution effect of CN is negative, an increase in the noise of performance measure IN (σ₁) would cause a reduction in the optimal incentive weight on CN.

Proof. If \( X < 0 \), then \( \frac{dB}{d\sigma^2} < 0 \).

Differentiating equations (12) and (13) with respect to \( \sigma^2 \), we obtain the following results

\[
\frac{dB_1}{d\sigma^2} = \alpha rq Y = \alpha \bar{Y} > 0, \quad (16)
\]

\[
\frac{dB_2}{d\sigma^2} = -r(\sigma^2 + q^2)\gamma = -\bar{Y} - r^2 \sigma^2 Y < 0, \quad (17)
\]

where \( \bar{Y} = rq^2 Y > 0 \) is related to the substitution effect. Hence we state the following proposition.

Proposition 3.
As the noise of performance measure CN increases (decreases), the optimal incentive weight on the IN increases (decreases) and the weight on the CN decreases (increases).

Comparing equation (10) with equation (9), we find that, other things being equal, an increase in \( \sigma^2 \) cause the risk of CN to be relatively higher than that of IN. That is, the agency cost of CN is relatively higher than that of IN. Therefore, the advertiser will place greater weight on the IN to replace CN, vice versa.

Differentiating equations (12) and (13) with respect to \( r \), we obtain the following results

\[
\frac{dB_1}{dr} = \alpha A - \sigma^2 B, \quad (18)
\]

\[
\frac{dB_2}{dr} = -A - r \sigma^2 S(Y + Z), \quad (19)
\]

where \( A = \frac{p^2 q^2 [h \sigma^2 - \sigma^3 (k - \alpha h)]}{H^2} \) is related to the substitution effect. The effect of changing Web publisher’s degree of risk aversion is ambiguous as is stated in the following proposition and corollaries.

Proposition 4.
As Web publisher’s degree of risk aversion \( r \) increases (decreases), the optimal incentive weight on IN and CN may either increase or decrease.

As \( r \) increases, the weight on IN and CN depends on the following two effects: (i) with Web publisher being more risk averse, advertiser must pay more risk premium to the Web publisher to satisfy Web publisher’s reservation utility (Equations (9) and (10)). This gives the advertiser an incentive to decrease the weight on IN and CN. (ii) it also depends on direction of substitution effect. If \( k \sigma^2 > h(\alpha \sigma^2 + \sigma^3) \) (\( k \sigma^2 < h(\alpha \sigma^2 + \sigma^3) \)), then \( A < 0 \) (\( A > 0 \)). That is, when the
advertising’s beneficial effect of IN’s uncertainty is larger (lower) than that of the CN, the advertiser will place greater weight on the CN (IN) to substitute for the IN (CN). Combining the two effects above, the increase in \( r \) may increase or decrease the weight on IN and CN, vice versa.

**Corollary 4.1.**

If the advertising’s beneficial effect of IN’s uncertainty is larger than that of CN, then an increase in Web publisher’s degree of risk aversion, \( r \), would cause a reduction in the optimal incentive weight on IN.

**Proof.** If \( k\sigma_1^2 > h(a\sigma_1^2 + \sigma_2^2) \), then \( A > 0 \) and \( \frac{d\beta}{dr} < 0 \).

**Corollary 4.2.**

If the advertising benefit of viewing the advertising per unit IN, being lower than that of interacting with the advertising per unit IN, causes the advertising’s beneficial effect of the IN’s uncertainty to be lower than that of the CN, an increase in Web publisher’s degree of risk aversion, \( r \), would cause the advertiser to place lower weight on the CN.

**Proof.** If \( k < ah \), then \( k\sigma_1^2 < h(a\sigma_1^2 + \sigma_2^2) \), \( A > 0 \) and \( \frac{d\beta}{dr} < 0 \).

If the advertising benefit of viewing the advertising per unit IN, being lower than that of interacting with the advertising per unit IN, \( k < ah \), causes the advertising’s beneficial effect of IN’s uncertainty to be lower than that of CN, \( k\sigma_1^2 < h(a\sigma_1^2 + \sigma_2^2) \), then \( A > 0 \). In this case, an increase in \( r \) (for example, a small Web publisher) would give the advertiser an incentive to place lower weight on the CN.

Differentiating equations (12) and (13) with respect to \( q \), we obtain the following results

\[
\frac{d\beta}{dq} = (rS + p^2)F = a\bar{F} + (r\sigma_1^2 + p^2)F > 0, \tag{20}
\]

\[
\frac{d\beta_1}{dq} = a\sigma_1^2F = \bar{F} > 0, \tag{21}
\]

where \( F = \frac{2r\sigma_1^2 q(rKS + kp^2)}{H^2} > 0 \), \( \bar{F} = a\sigma_1^2F > 0 \). Given these results, we propose the following:

**Proposition 5.**

As the exogenous factor of IN, \( q \), increases (decreases), both the optimal incentive weights on IN and CN increase (decrease).

If the exogenous factor of IN, \( q \), is higher (for example, the Internet is more popular than before), Web publisher’s effort of mass marketing would attract more consumers landing on the Website. Hence, more consumers would interact with the advertising by the BC. In this case, the incentive based on IN and CN would induce more mass marketing effort of Web publisher, therefore, the optimal incentive weight on IN and CN would be higher, and vice versa.

Differentiating equations (12) and (13) with respect to \( p \), we obtain the following results

\[
\frac{d\beta}{dp} = -aq^2G = -a\bar{G}, \tag{22}
\]

\[
\frac{d\beta_1}{dp} = (q^2 + r\sigma_1^2)G = \bar{G} + r\sigma_1^2G, \tag{23}
\]

where \( G = \frac{2r[p\sigma_1^2 q^2(\sigma_1^2 + k)(r\sigma_1^2 S + \sigma_2^2 q)]}{H^2}, \bar{G} = q^2G \). Note that \( \bar{G} \) is related to the substitution effect. The effect of changing CN’s exogenous factor is ambiguous as stated in the following proposition and corollary.

**Proposition 6.**

As the exogenous factor of CN, \( p \), increases (decreases), the optimal incentive weights on IN and CN may either increase or decrease.

As \( p \) increases (decreases), for example, an increases (decreases) in the advertiser’s reputation, the weights on IN and CN depends on the relatively advertising benefit between the viewing the advertising and interacting with the advertising per unit IN as stated in the following corollary.
Corollary 6.1.

If the advertising benefit of viewing the advertising per unit IN is larger (lower) than that of interacting with the advertising per unit IN, then an increase in the exogenous factor of CN, p, would cause the advertiser to place larger (lower) weight on IN and lower (larger) weight on CN.

Proof. If \( k > ah \), then \( G < 0 \) and \( H > 0 \), therefore, \( \frac{d\beta_1}{dp} > 0 \) and \( \frac{d\beta_2}{dp} < 0 \).

If \( k < ah \), then \( G > 0 \) and \( H < 0 \), therefore, \( \frac{d\beta_1}{dp} < 0 \) and \( \frac{d\beta_2}{dp} > 0 \).

As the exogenous factor of CN, p, is greater, Web publisher’s effort of target marketing would attract more consumers interacting with Website. If the advertising benefit of viewing the advertising per unit IN is larger than that of interacting with the advertising per unit IN or \( k > ah \), the advertiser would place larger weight on IN and lower weight on CN, and vice versa.

Differentiating equations (12) and (13) with respect to \( k \) and \( h \) we obtain the following results

\[
\frac{d\beta_1}{dk} = q^2(rS + p^2) = \frac{H}{d} + \frac{q^2(r\sigma_e^2 + p^2)}{H} > 0, \tag{24}
\]

\[
\frac{d\beta_2}{dk} = I > 0, \tag{25}
\]

\[
\frac{d\beta_1}{dh} = \frac{r\alpha S q^2}{H} > 0, \tag{26}
\]

\[
\frac{d\beta_2}{dh} = \frac{r\sigma_e^2(\alpha^2 q^2 + p^2) + p^2 q^2}{H} = \frac{d}{H} + \frac{p^2(r\sigma_e^2 + q^2)}{H} > 0. \tag{27}
\]

where \( I = \frac{\alpha q^2 r\sigma_e^2}{H} > 0 \). Given these comparative static results, we propose the following:

Proposition 7.

As the marginal advertising benefit of viewing the advertising by the consumers landing on the Website, \( k \), increases (decreases), both the optimal incentive weights on IN and CN increase (decrease).

Proposition 8.

As the marginal advertising benefit of interaction by the consumers who interact with the advertising, \( h \), increases (decreases), both the optimal incentive weights on IN and CN increase (decrease).

CONCLUSIONS

The objective of this paper is to set up an online branding advertising pricing model including the strategic interdependence between advertisers and Web publishers to investigate the optimal advertising pricing for advertisers.

It is hoped that this study makes two contributions. First, built on the previous work by Holmstrom (1979), Hauser, et al. (1994), Feltham and Xie (1994), and Hemmer (1996), we expand our theoretical framework that emphasizes the strategic interdependence between Web publisher and advertiser, specifically, using linear correlation to describe the relation between the performance measures, IN and CN. Second, it provides a unique insight into optimal incentive weights on the performance measures of advertiser’s online branding advertising pricing, and it characterizes the value of providing additional performance measure, CN.

These results have interesting implications for advertisers who attempt to sign online branding advertising pricing contract with Web publishers. First, adding clicks number as an additional performance measure of performance-based pricing scheme would improve the advertiser’s net advertising benefit (the effect of lift in brand awareness) because it provides diverse performance measure that can be used to induce actions more congruent with the advertiser’s benefit. Payment based only on CPM could not induce the effort of Web publisher to reach as close to advertiser’s target consumers as he wishes. And hence it cannot achieve the advertising benefit of advertiser.
Second, the payments based on impressions number and clicks number act as substitute to induce optimal effort levels of Web publisher. That is, if either noise is relatively high the optimal incentive weight suggests that the advertiser should place greater emphasis on the substitute performance measure in order to lower agency cost.

Third, facing a smaller Web publisher, the advertiser should place lower weight on the clicks number when the advertising benefit (the effect of lift in brand awareness) of viewing the advertising per unit IN (impressions number) is lower than that of interacting with the advertising per unit IN, and vice versa.

Forth, the advertiser should increase in the weight on impressions number and clicks number when the Internet is more popular.

Fifth, an advertiser with finer reputation should place larger weight on impressions number and lower weight on clicks number when the advertising benefit (the effect of lift in brand awareness) of viewing the advertising per unit IN is larger than that of interacting with the advertising per unit IN, and vice versa.

Finally, the advertiser should place higher (lower) weight on the impressions number and clicks number when the marginal advertising benefit (the marginal effect of lift in brand awareness) of viewing the advertising by the consumers landing on the Website is higher or the marginal advertising benefit of interaction with the consumers is higher, and vice versa.

APPENDIX A

The advertiser’s maximization problem when he uses only IN as the single measure of the performance-based pricing scheme can be expressed as:

\[
\max_{\tilde{\beta}} \tilde{\Pi} = \mathbb{E}[u(n_i) + u(n_j) - \tilde{w}],
\]

s.t. \( E[1 - e^{-r(\tilde{w} - \alpha \tilde{\beta})}] \geq 0 \), \( \Theta, \Phi \in \arg \max_{\alpha, \beta} E[1 - e^{-r(\tilde{w} - \beta \alpha \tilde{\beta})}] \) \hspace{1cm} (IR)

where \( \tilde{\Pi} \) is advertiser’s net advertising benefit when he uses only IN as the single measure of the performance-based pricing scheme. \( \tilde{\beta} \) is the price paid per unit of performance measure IN. \( \tilde{w} \) is the total fee advertiser pays to Web publisher, \( \tilde{\beta} = \tilde{\beta}_1 \).

Substituting equations (3), (4), and (5) into the CE of Web publisher and differentiating with respect to \( \Theta \) and \( \Phi \) yields the Web publisher’s optimal efforts choices. Here:

\[
\Theta' = \tilde{\beta}q,
\]

\[
\Phi' = 0.
\]

(A1)

(A2)

The advertiser’s first-order condition may be used to obtain the optimal price paid per unit of IN:

\[
\tilde{\beta}' = \frac{q^2K}{q^2 + r\sigma_i^2}.
\]

(A3)

Substituting first-order condition into \( \tilde{\Pi} \) yields the Web publisher’s optimal net advertising benefit when he uses only IN as the single measure of the performance-based pricing scheme or:

\[
\tilde{\Pi}' = \frac{q^2K^2}{2(q^2 + r\sigma_i^2)}.
\]

(A4)

Substituting equations (1), (2), (3), (4), (7), (8), (9), (10), (11), (12) and (13) into \( \Pi \) yields the Web publisher’s optimal net advertising benefit when he uses both IN and CN as performance measures of the performance-based pricing scheme or:

\[
\Pi' = \frac{J}{2H},
\]

where \( J = q^2(h^2\rho^2 + q^2K^2(p_i^2 + rS)) + r\sigma_i^2(aKq^2 + hp^2) \).

Subtracting equation (A4) from equation (A5), we obtain
\[ \Pi' - \hat{\Pi}' = \frac{\hat{Y}^2}{2(q^2 + \sigma_Y^2)H} > 0, \] 

where \( \hat{Y} = HY \).

REFERENCES


