Trading on Information About Noise in Financial Markets

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ABSTRACT

Many investors in financial markets are relatively inexperienced. Much of the literature focuses on the incentives of sophisticated informed agents to produce information about future cash flows, and not on agents that attempt to acquire information about the trades of naïve agents. However, unsophisticated agents are important aspects of financial markets and worth analyzing further. In this paper, we provide a theoretical perspective that addresses trading based on private information about the trades of naïve investors. We analyze the effect of such information on liquidity costs, price efficiency, and volatility. We show that the optimal acquisition of information about noise traders balances the extent of informed trading and the direct benefits from noise acquisition. When management possesses information about the activities of noise traders, the optimal disclosure strategy of management depends on whether existing shareholders are helped or harmed by enhanced informational efficiency. When shareholders own shares in private firms, disclosure is optimal to allow private firms to allocate resources more efficiently. On the other hand, when private firms are acquisition targets managers prefer to not disclose noise trades to lower the acquisition price.

INTRODUCTION

A significant branch of literature on financial markets focuses on asymmetric information between market participants. In this research, it is most common to analyze agents which have information about future cash flows. These better informed agents interact with uninformed agents in a rational expectations equilibrium. To prevent the system from fully revealing prices, and thereby to preserve incentives to acquire information, one injects some exogenous noise, often interpreted as supply shocks or liquidity trading. Examples of such papers are Kyle (1985) and Grossman and Stiglitz (1980).

If one takes the existence of noise traders seriously, it is sensible to allow for the possibility that other agents might acquire information about this component of demand. Indeed, the way these noise traders have been interpreted in the literature suggests that gathering information about their activities might be an important feature of financial markets. For example, if one interprets them as retail traders expressing their demand through mutual funds, information about retail demand is in possible to obtain by analyzing patterns in mutual fund flows. Further, managers may possess information about the level of shareholdings in their company as well as how shareholdings change over time, which is another measure of retail demand.

Another way to obtain information about retail flows is to recognize that investors often share commonalities in crude heuristics. For example, a investors often use technical analysis methods that do not receive much support in the data (such as “head and shoulders”). Industry or sector fashions and fads, and populist theories of the stock market tend to propagate themselves quite commonly (Shiller (2000)). Investors commit simple errors, such as confusing ticker symbols or reacting to already-public information (see Rashes (2001), Ho and Michaely (1988), and Huberman and Regev (2001)). Further, the
work of Barber and Odean (2001), Brennan (1995), and Odean (1998, 1999) suggests that individual investors trade excessively due to overconfidence, have subpar investment performance, and are also susceptible to aspects such as loss aversion, indicating imperfect rationality.

There are also examples of naïve heuristics propagated by investment analysts. Thus, in the 1990s tech firms were valued based upon sales and ``eyeballs'' rather than earnings, which \textit{ex post} turned out to be inappropriate. With the rise of the mass media, it is also easier for dubious stock market information to be spread rapidly and widely, and information about such theories and heuristics is potentially available with some investigation.

There are thus many potential ways to obtain information about the activities of informationless, naïve traders. Note that such information is a valuable part of investment analysis since knowing whether such investors will buy or sell allows for better forecasting of cash flows from market prices (which are correlated with the trades of naïve investors).

Motivated by the above observations, in this paper, we analyze a setting where a class of agents acquires information about future cash flows, and another class gathers information about the noise in the aggregate demand. We are able to solve for an analytic solution to this model, and analyze comparative statics. We find that in fact, an increase in the mass of cash-flow informed agents can increase the expected utility from acquiring information about noise. On the other hand an increase in the mass of noise-informed agents reduces returns to trading on fundamental information.

We extend the model by analyzing incentives for the public firm to disclose the amount and sign of noise trading in a setting where there is a private firm within the same industry sector. The incentives for disclosure depend on the strategic interaction between the public firm and a private firm that allocates resources based on the public firm's stock price. More efficient prices lead to better resource allocation and an enhanced value of the private firm. Hence, the incentives to disclose depend on whether the public firm's shareholders benefit or harmed by an increased value of the private firm. For example, if the public firm's shareholders also own shares in the private firm, disclosure is optimal. On the other hand, if the public firm intends to grow via acquiring related companies, then it has an incentive to withhold information from private firms. Doing so lowers the private firm's value and thus reduces its acquisition price.

Our work can also be motivated by Kyle (1984), who suggests modeling agents that acquire information about the activities of informationless traders. The paper is most closely related to that of Lindsey (1990) who analyzes a model where the market maker knows the variance of noise trades. In contrast we model a class of competitive agents that possesses information about the realization of noise trades.

This paper is organized as follows. Section 2 presents the basic model. Section 3 analyzes the equilibrium where agents possess information about cash flows as well as noise trades. Section 4 analyzes the expected utilities from trading on cash flow and noise information. Section 5 analyzes the incentives to disclose information about noise trades. Section 6 concludes. Proofs appear in the Appendix.

**THE ECONOMIC SETTING**

We consider a model of a financial market where the security pays off an amount $F$ which equals the sum of two independent components, $F \equiv \theta + \epsilon$. Both $\theta$ and $\epsilon$ have ex ante means of zero and are normally distributed with variances $\nu_\theta$ and $\nu_\epsilon$, respectively. Risk-neutral, competitive uninformed agents set the price conditional on all available public information.
We consider trades that are uncorrelated with future cash flows. These might either arise due to unanticipated liquidity needs or due to naïve heuristics used by unsophisticated agents. We propose that there are two components of noise trades, \( z_1 \) and \( z_2 \). Each class is independent and identically distributed as \( N(0, \sigma^2) \).

Informed agents observe the realization of \( \theta \) exactly. We also model a class of traders with information about \( z_1 \), a component of noise trades. These agents condition on the market price as well as \( z_1 \) to construct their demand. We refer to traders with information about \( \theta \) as cash flow informed traders and those with information about \( z_1 \) as noise informed traders. Both cash flow informed and noise informed traders have negative exponential utility with risk aversion \( R \). All traders are competitive price-takers, and the total masses of cash flow informed and noise informed agents are \( m \) and \( n \), respectively.

Due to the existence of a risk-neutral, competitive, market-making sector, the equilibrium price in this setting is defined as the expectation of \( \theta \) conditional on the market price, or, equivalently, the net demands of all of the agents.

**EQUILIBRIUM**

Let \( x \) denote the demand of each cash flow informed agent and \( y \) denote the demand of each noise informed agent. Since agents have negative exponential utility, standard arguments indicate that

\[
x = \frac{\theta - P}{Rv}. \tag{1}
\]

The total demand is then

\[
m(\theta - P) + ny + z_1 + z_2.
\]

In equilibrium, as we will see, the price is a linear function of net demand. Since the noise informed agents know their own demand and the market price, they condition on the variables \( \kappa \) and \( z_1 \), where \( \kappa \) is given by

\[
\kappa = \frac{m\theta}{Rv} + z_1 + z_2. \tag{2}
\]

However, since \( z_1 \) is a nuisance variable, it is easy to show that conditioning on \( \kappa \) and \( z_1 \) is equivalent to conditioning on the single variable

\[
\tau = \theta + \frac{Rvz_2}{m}.
\tag{3}
\]

Let \( \mu \equiv E(\theta \mid \tau) \) and \( \nu \equiv var(\theta \mid \tau) \). Then, we have that

\[
y = \frac{\mu - P}{Rv}. \tag{4}
\]

Now, the market maker observes the net demand, which equals:

\[
m(\theta - P) + n(\mu - P) + z_1 + z_2,
\]

or,

\[
m(\theta - P) + n(\mu - P) + z_1 + z_2.
\]
which is equivalent to conditioning on
\[ \tau_m = \frac{m\theta}{Rv} + \frac{n\mu}{Rv} + z_1 + z_2. \]  

(5)

Since the market maker earns zero expected profits, we have that
\[ P = E(\theta \mid \tau_m). \]

Fortunately, \( \tau_m \) can be expressed solely in terms of exogenous variables, which allows us to solve for the price \( P \) in closed form.

The following proposition provides the equilibrium solution to the market price.

**Proposition 1:** The price \( P \) is given by
\[ P = H_0 \theta + H_1 z_1 + H_2 z_2, \]

where
\[ H_0 = \frac{m^2 v_\theta [m^2 v_\theta + mnv_\theta + R^2 v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)]}{D} \]

(6)

\[ H_1 = \frac{mRv_\varepsilon v_\theta [m^2 v_\theta + mnv_\theta + R^2 v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)] [m^2 v_\theta + R^2 v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)]}{D} \]

(7)

\[ H_2 = \frac{mRv_\varepsilon v_\theta [m^2 v_\theta + mnv_\theta + R^2 v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)]^2}{D} \]

(8)

\[ D = v_\theta^3 (m^6 + 2m^5 n) + m^4 v_\theta^2 [n^2 v_\theta + 2R^2 v_\varepsilon v_\varepsilon (2v_\varepsilon + v_\theta)] \]

(9)

\[ + 2m^2 nRv_\varepsilon v_\theta^2 v_\varepsilon (2v_\varepsilon + v_\theta) + m^2 R^2 v_\varepsilon^2 v_\theta v_\varepsilon [n^2 v_\theta + R^2 v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)(5v_\varepsilon + v_\theta)] \]

\[ + 2mnR^4 v_\varepsilon^4 v_\theta v_\varepsilon^2 (v_\varepsilon + v_\theta) + 2R^6 v_\varepsilon^4 v_\varepsilon^2 (v_\varepsilon + v_\theta)^2. \]

(10)

The behavior of the derivatives of \( H_0 \), \( H_1 \), and \( H_2 \) with respect to \( m \) and \( n \) is formally described in the appendix. It can easily be verified that \( H_0 \) is increasing in \( m \) and \( n \). The trades of both noise informed and cash flow informed traders are correlated with \( \theta \), which causes the weight of \( \theta \) to increase in \( m \) as well as \( n \).

We also find that \( H_1 \) is decreasing in \( n \) but \( H_2 \) is increasing in \( n \). This is because the trades of noise informed traders are correlated with \( z_2 \) but uncorrelated with \( z_1 \), as discussed above. An increase in \( n \) therefore increases the coefficient of \( z_2 \) in the Bayesian inference, and, in turn, decreases the coefficient on \( z_1 \). The coefficients \( H_1 \) and \( H_2 \) can be increasing or decreasing in \( m \).

An increase in \( m \) increases the covariance of the net order with \( \theta \) but also increases the total variance of the orders. The net effect therefore is ambiguous. The appendix argues that generally, when \( m \) is low, the covariance between net demand and \( \theta \) is weak, and an increase in \( m \) has a big effect on the covariance, leading \( H_1 \) and \( H_2 \) to increase in \( m \). When \( m \) is high, the net demand is already very informative and the variance effect dominates.

We now turn to the volatility of the price. It is easily shown that
\[ \text{var}(P) = H_0^2 v_\theta^2 + (H_1^2 + H_2^2) v_\varepsilon = \frac{m^2 v_\theta^2 [m^2 v_\theta + mnv_\theta + R^2 v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)]^2}{D}. \]

The appendix shows that the volatility of the price is increasing in both \( m \) and \( n \). An increase in
either type of trader enhances price fluctuations, resulting in increased price volatility. It is also shown in the appendix that, interestingly, the volatility of the price is decreasing in $v_{z}$. An increase in the variance of noise trading decreases the informativeness of net demand, reducing price fluctuations.

Next, we examine the efficiency of the stock price, $[\text{var}(\theta + \varepsilon \mid P)]^{-1}$. The following proposition can easily be derived.

**Proposition 2:** The efficiency of the stock price, $[\text{var}(\theta + \varepsilon \mid P)]^{-1}$, is given by

$$\text{var}(\theta + \varepsilon \mid P))^{-1} = \frac{D}{B},$$

where

$$B = v_{\varepsilon}\{v_{\varepsilon}^{2}(m^{2} + 2m^{2}n) + m^{4}v_{\varepsilon}^{2}\{n^{2}v_{\theta} + 4R^{2}v_{\varepsilon}v_{z}(v_{\varepsilon} + v_{\theta})\} + 4m^{3}nR^{2}v_{\varepsilon}v_{\theta}v_{z}(v_{\varepsilon} + v_{\theta}) + 2m^{2}R^{2}v_{\varepsilon}v_{\theta}v_{z}(v_{\varepsilon} + v_{\theta})\{n^{2}v_{\theta} + 5R^{2}v_{\varepsilon}v_{z}(v_{\varepsilon} + v_{\theta})\} + 2mnR^{4}v_{\varepsilon}^{2}v_{\theta}v_{z}^{2}(v_{\varepsilon} + v_{\theta})^{2}2R^{6}v_{\varepsilon}^{3}v_{z}^{3}(v_{\varepsilon} + v_{\theta})^{3}\}].$$

The informational efficiency of the price is increasing in both $m$ and $n$, the masses of agents informed about cash flows and noise, respectively. Thus, an increase in the mass of cash flow informed and noise-informed agents increases price efficiency because the net demand is more informative in the presence of these agents. Specifically, the cash flow informed and noise informed agents together provide a more accurate signal of $\theta$, making the price more efficient.

We finally turn to the liquidity costs in our model. These costs are the trading losses incurred by the two classes of noise traders and are given by

$$\phi = E[z_{1}(P - \theta)] + E[z_{2}(P - \theta)] = (H_{1} + H_{2})v_{z}.$$ 

Substituting for $H_{1}$ and $H_{2}$ from (8) and (9), respectively, we have:

$$\phi = mRv_{\varepsilon}v_{\theta}v_{z}[2m^{2}v_{\varepsilon}^{2} + mnv_{\theta} + 2R^{2}v_{\varepsilon}v_{z}(v_{\varepsilon} + v_{\theta})[m^{2}v_{\theta} + mnv_{\theta} + R^{2}v_{\varepsilon}v_{z}(v_{\varepsilon} + v_{\theta})]]D.$$ 

We then have the following proposition.

**Proposition 3:** The expected losses of $z_{1}$ traders decrease in $n$. The expected losses of $z_{2}$ traders increase in $n$. The net effect of an increase in $n$ on the aggregate expected losses of noise traders is ambiguous.

The intuition for the above result is that the $n$ traders' conditioning variable $\tau$ (from (3) is a linear combination of $\theta$ and $z_{2}$ and does not involve $z_{1}$. Thus, as $n$ increases, the weight placed on $z_{2}$ in the price increases. In the Bayesian conditioning by the market maker, consequently, the weight placed on $z_{1}$ decreases. The appendix argues that when $n$ and $m$ are low a marginal increase in $n$ has a big effect on the informativeness of the net demand and the former effect dominates the latter. For large $n$ and $m$ the reverse is true.

### GAINS FROM TRADE FOR CASH FLOW INFORMED AND NOISE INFORMED AGENTS

We begin by stating the following lemma, which is a standard result on multivariate normal random variables (see, for example, Brown and Jennings, 1989).

**Lemma 1:** Let $Q(\chi)$ be a quadratic function of the random vector $\chi$: $Q(\chi) = C + B'\chi - \chi'A\chi$.
where $\chi \sim N(\mu, \Sigma)$, and $A$ is a square, symmetric matrix whose dimension is the same as that of $\chi$.

We then have

$$E[\exp(Q(\chi))] = |\Sigma|^{-\frac{1}{2}} 2A + \Sigma^{-1} |^\frac{1}{2} \times$$

$$\exp \left( C + B^T \mu + \mu^T A \mu + \frac{1}{2} (B^T - 2\mu^T A) (2A + \Sigma^{-1})^{-1} (B - 2A\mu) \right).$$

(12)

The ex ante utility of the agents is derived by an application of Lemma 1. Define $\lambda = [\theta \in z_1 z_2]$ and let $\Sigma$ denote the variance matrix for this vector. Then, we can construct the square, symmetric matrix $A$ such that $RW = \lambda^T A \lambda$, where $W$ is the wealth of the agent. Noting then that the ex ante expected utility is given by $EU = E[-\exp(-RW)]$, we can apply Lemma 1 with $\mu = 0$, $C = 0$, and $B = 0$. The agent's ex ante utility thus becomes

$$EU = E[-\exp(-\lambda A \lambda^T)] = -|\Sigma|^{-\frac{1}{2}} 2A + \Sigma^{-1} |^{-\frac{1}{2}} = -|2A\Sigma + I|^{-\frac{1}{2}}.$$  

(13)

We denote the determinant $|2A\Sigma + I|$ as $Det_I$ for cash flow informed agents and as $Det_N$ for noise informed agents, and their respective expected utilities as $EU_I$ and $EU_N$, respectively.

Note that the expected utilities are monotonically increasing in the corresponding determinants.

And, we then have the following proposition.

**Proposition 4:** The expected utility from being informed about cash flows is a monotonic transformation of $Det_I$, where

$$Det_I = C/D$$

and, in turn, where

$$C = v_\theta^3 (m^6 + 2m^5 n) + m^4 v_\theta^2 [n^2 v_\theta + 4R^2 v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)]$$

$$+ 4m^3 nR^2 v_\varepsilon v_\theta^2 v_\varepsilon (v_\varepsilon + v_\theta) + m^2 R^2 v_\varepsilon^2 v_\theta^2 v_\varepsilon (v_\varepsilon + v_\theta) [n^2 v_\theta + 5R^2 v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)]$$

$$+ 2mnR^4 v_\theta^2 v_\varepsilon^2 (v_\varepsilon + v_\theta)^2 + 2R^6 v_\varepsilon^3 v_\theta^3 (v_\varepsilon + v_\theta)^3.$$  

(15)

The expected utility from being informed about noise is a monotonic transformation of $Det_N$, where

$$Det_N = E/D$$

and, in turn, where

$$E = v_\theta^3 (m^6 + 2m^5 n) + m^4 v_\theta^2 [n^2 v_\theta + v_\varepsilon v_\varepsilon (2R^2 (2v_\varepsilon + v_\theta) + 2Rv_\theta - v_\theta)]$$

$$+ 2mnR^4 v_\theta^3 v_\varepsilon^3 (2v_\varepsilon + v_\theta) + m^2 R^2 v_\varepsilon^2 v_\varepsilon^2 v_\varepsilon (v_\varepsilon + v_\theta) (R^2 (5v_\varepsilon + v_\theta) + 2Rv_\theta - v_\theta)^2$$

$$+ 2mnR^3 v_\theta^3 v_\varepsilon v_\varepsilon^2 (v_\varepsilon + v_\theta) + 2R^6 v_\varepsilon^3 v_\theta v_\varepsilon^2 (v_\varepsilon + v_\theta)^2.$$  

(17)

Note that the expected utility in (13) exists only when $Det_I$ and $Det_N$ are positive. $Det_I$ is always positive, but $Det_N$ can be positive or negative. To ensure the expected utility exists of noise informed traders exists, we will henceforth assume that $R > 0.5$, which is a sufficient condition for $Det_N$ to be positive.

We then have the following proposition.

**Proposition 5:**

1. The expected utility from trading on information is decreasing in $n$, the mass of agents informed about noise.

2. The expected utility from trading on noise can be increasing or decreasing in $m$, the mass of agents.
informed about future cash flows.

3. The product of $\text{Det}_t$ and the informational efficiency of the stock price equals $\nu^{-1}$, an exogenous constant.

4. The expected utility of trading on cash flow information decreases in the mass of cash flow informed agents, $m$, and the expected utility of trading on noise information decreases in the mass of noise informed agents, $n$.

We now discuss the four parts of the proposition. The first part indicates that the presence of noise informed agents makes the cash flow informed agents worse off. This is simply because the noise informed traders, in effect, compete with cash flow informed traders, and cause the net demand to become more informative, cutting into the trading benefits of cash flow informed traders.

The second part can be explained as follows. Note first that cash flow informed traders are necessary for noise informed traders to earn any rents at all from trading. This is because if the net demand is not informative about cash flows, market makers will not condition on the net demand, making noise information unprofitable. However, too many traders on fundamental information cut into the trading benefits of noise informed traders. Generally, for small $m$, increasing $m$ increases the expected utility of noise informed traders, whereas for large $m$ the opposite is true.

The third part of the proposition recalls Grossman and Stiglitz in arguing that the expected utility from trading on cash flow information is inversely related to the efficiency of the stock price. The more efficient the stock price, the less the benefit from trading on cash flow information.

Finally, the fourth part of the proposition provides the intuitive result that increasing the mass of agents with information of a certain type reduces the expected utility of trading on that type of information.

The number of noise informed traders can be endogenized by assuming that they can acquire information about $z_1$ at a cost of $c$. Note that agents who do not acquire the information (i.e., who choose to be uninformed) will choose not to trade since they are risk averse and the price, being set by risk-neutral, uninformed market makers will not offer them a risk premium (i.e., the numerator in (4) equals zero for an informed agent without any informational advantage over the market maker, implying no trade). Therefore, the equilibrium number of noise informed traders is given by the solution to

$$EU_N = -\frac{1}{\text{Det}_N^{0.5}} = \exp(-Rc)$$

or

$$\exp(2Rc) = \text{Det}_N.$$ 

We provide a numerical example where all parameters are set to unity except that $c = 0.005$. Consider the case where $m = 0.3$. Then, the equilibrium level of $n$ is 5.13, and the efficiency of stock prices is 1.07. When $m$ rises to 0.4, $n$ rises to 7.26 and the efficiency of stock prices rises to 1.15. Now suppose that $m = 2$. In this case the equilibrium $n$ is 7.84, and stock price efficiency is 4.72. When $m$ rises to 2.2, $n$ drops to 7.61 while efficiency still rises to 6.73. This illustrates that the effect of $m$ on the equilibrium $n$ depends on the level of $m$.

### INCENTIVES FOR DISCLOSURE

Suppose that the public firm also possesses private information about $z_1$. This information can accrue via analyzing patterns of shareholdings or trades. We now consider if the firm has an incentive to
disclose information about \( z_1 \). Of course, the firm also has an incentive to disclose cash flow information. Since the incentives to disclose cash flow information have already been analyzed in the literature starting with Diamond (1985), we assume that information about \( \theta \) is produced by outside analysts, and focus on the disclosure of noise information. We analyze disclosure of such information by considering a private firm whose profits are correlated with that of the private firm. Specifically, consider a privately held firm with the following profit function:

\[
\pi = K \theta - 0.5 K^2.
\]

The firm chooses capital \( K \) to maximize its expected profits conditional on the market price of the public firm. This implies that

\[
K = E(\theta \mid P) = P.
\]

Thus, the maximized profits equal

\[
\pi = \theta P - 0.5 P^2.
\]

The unconditional expectation of these profits equals

\[
\alpha = H_0 v_\theta - 0.5 \text{var}(P) = v_\theta (H_0 - 0.5 H_0^2) - 0.5 v_\theta (H_1^2 + H_2^2).
\]

Substituting for \( H_0, \ H_1, \) and \( H_2 \) from (7)-(9), we have

\[
\alpha = \frac{m^2 v_\theta^2 [m^2 v_\theta^2 + mn v_\theta + R^2 v_\epsilon^2 v_z (v_\epsilon + v_\theta)]^2}{2D}.
\]

This brings us to the following proposition.

**Proposition 6:** The expected value of the private firm is increasing in \( m \), the mass of cash flow informed traders, as well as in \( n \), the mass of noise informed traders.

Thus, the private firm benefits from having both types of informed traders in the market for the public firm.

We now focus on the public firm's incentives to increase or lower the value of the private firm. We assume that the public firm can costlessly disclose the signal \( z_1 \). If the signal is in fact disclosed, then the price \( P \) is given by

\[
P = E(\theta \mid \tau)
\]

where \( \tau \) is given by (3). As shown in the proof of Proposition 1 within the Appendix, in this case,

\[
E(\theta \mid \tau) = a_0 \theta + a_2 z_2
\]

where

\[
a_0 = \frac{m^2 v_\theta}{m^2 v_\theta + R^2 v_\epsilon^2 v_z}.
\]

\[
a_2 = \frac{m R v_\epsilon v_\theta}{m^2 v_\theta + R^2 v_\epsilon^2 v_z}.
\]

The expected value of the private firm now becomes:

\[
\alpha_d = \frac{m^2 R^2 v_\epsilon^2 v_\theta^2 v_z [m^4 v_\theta^2 + 2 m^2 R^2 v_\epsilon^2 v_\theta v_z (v_\epsilon + v_\theta) + R^4 v_\epsilon^2 v_z^2 (v_\epsilon + v_\theta)^2]}{2D (m^2 v_\theta + R^2 v_\epsilon^2 v_z)} > 0.
\]
We consider two scenarios. In the first scenario, we assume that shareholders in the public firm also hold shares in the private firm. In this instance, assume that the public firm acts in the interest of existing shareholders. Then, the objective of the public firm is to maximize the value of the private firm's shares. We then have the following proposition.

**Proposition 7:** If the public firm's shareholders also own shares in the private firm, then the optimal strategy of the public firm is to disclose $z_1$, the amount of the noise trade.

Next, suppose the private firm is a potential target of acquisition by the private firm. In this case, we assume that the public firm obtains a synergy benefit of $A_i$ from acquiring the private firm, and retains the original manager to run the firm. Further, the cash flows from the private firm accrue after the assets of the public firm have paid off and the shareholders in the public firm do not trade claims in the private firm after acquisition. Finally, the public firm precommits to a disclosure policy and the manager of the private firm uses the information conveyed by the disclosure policy whether or not the private firm is acquired.

Let us define the unconditional mean of the private firm's value as $\beta$, with $\beta = \alpha$ without disclosure and $\beta = \alpha_d$ with disclosure. There are two potential bidders, one of them being the public firm and another a private firm with synergy benefits $A_i < A$. Owing to Bertrand competition between bidders, the competing bidder firm bids the unconditional expected value of $\beta + A_i$ for the private firm. To win the bid, therefore, the public firm has to match $\beta + A_i$. The public firm pays for the acquisition in cash, but the cash balance is limited, and borrowing is not permitted. The cash balance is denoted by $C$. This implies the following disclosure strategy:

**Proposition 8:**
1. If $\alpha_d + A_i > C > \alpha + A_i$, then the optimal strategy of the firm is not to disclose $z_1$.
2. If $C < \alpha + A_i$ or $C > \alpha_d + A_i$, then the firm is indifferent between disclosing and not disclosing $z_1$.

The idea is that the public firm may choose not to disclose $z_1$ to lower the acquisition price. Overall, the disclosure policy of the firm depends on whether the public firm benefits or is hurt by an increased value of the private firm's share via disclosure.

**CONCLUSION**

Many agents in financial markets are relatively inexperienced and likely lack financial sophistication. Indeed, many models, such as those based on Kyle (1985) or Grossman and Stiglitz (1980), simply assume the existence of unsophisticated agents, and instead focus on the incentives of presumably sophisticated informed agents to produce information. However, recent research in finance (e.g., Odean (1998, 1999)) indicates that unsophisticated agents are important aspects of financial markets and are subject to common, but mistaken, heuristics.

In this paper, we provide a theoretical perspective that solves for an equilibrium where some agents trade on information about noise trades. We show that the existence of such agents improves market efficiency but that they discourage the production of fundamental information. On the other hand, the existence of agents informed about cash flows encourages the collection of information about noise.

We extend the model to a setting where managers can disclose the extent of noise trading. The incentives for disclosure depend on the strategic interaction between the public firm and a private firm.
that allocates resources based on the public firm's stock price. More efficient prices lead to better resource allocation and an enhanced value of the private firm. Hence, the incentives to disclose depend on whether the public firm's shareholders benefit or harmed by an increased value of the private firm. For example, if the public firm's shareholders also own shares in the private firm, disclosure is optimal. On the other hand, if the public firm intends to grow via acquiring related companies, then it has an incentive to withhold information from private firms. Doing so lowers the private firm's value and thus reduces its acquisition price.

We believe that our work is a first consideration of the issues surrounding naïve investors, and our work suggests a fertile agenda for future research. For example, dynamic models might be a natural extension. Whether such agents create excessive price fluctuations would also be interesting to investigate. These and other issues are left for future research.

REFERENCES


Proof of Proposition 1: Note that

$$P = \frac{\text{cov}(\theta, \tau_m)}{\text{var}(\tau_m)} \tau_m.$$  

(18)

Now, let $\mu = a_0 \theta + a_2 z_2$. We then have

$$\text{cov}(\theta, \tau_m) = \frac{v_\theta (a_0 n v_e + m v)}{R v v_e},$$

(19)

and

$$\text{var}(\tau_m) = a_0^2 n^2 v_e^2 v_\theta + 2 a_0 m n v_e v_\theta + a_2^2 n^2 v_e^2 v_\theta + 2 a_2 n R v e v_e v_\theta + m^2 v^2 v_\theta + 2 R^2 v^2 v_e^2 v_z.$$  

(20)

Note, however, that

$$a_0 = \frac{m^2 v_\theta}{m^2 v_\theta + R^2 v_e^2 v_z},$$

(21)

and

$$a_2 = \frac{m R v_e v_\theta}{m^2 v_\theta + R^2 v_e^2 v_z}. $$

(22)

Also,

$$v = v_e + v_\theta - \left[\frac{\text{cov}(\theta, \tau)}{\text{var}(\tau)}\right]^2 = v_e + \frac{R^2 v^2 v_\theta v_z}{m^2 v_\theta + R^2 v_e^2 v_z}.$$  

(23)

Substituting for $a_0$, $a_2$, and $v$ from (21)-(23) into (19) and (20) above and substituting for $\text{cov}(\theta, \tau_m)$ and $\text{var}(\tau_m)$ into (18) completes the proof. ||
Behavior of the Derivatives of $H_0$ with respect to $m$ and $n$:

From (7), we have that

\[
\begin{align*}
D^2 \frac{dH_0}{dm} &= 2mR^2v_e^2v_\theta v_z (2m^6v_\theta^3 + 3m^4nv_\theta^3 + 3m^4v_\theta^2(n^2v_\theta + 2R^2v_e v_z (v_e + v_\theta) + v_\theta)) + m^3nv_\theta^2(n^2v_\theta + 8R^2v_e v_z (v_e + v_\theta)) + 3m^2R^2v_e v_\theta v_z (v_e + v_\theta)(n^2v_\theta
\]
\[+ 2R^2v_e v_\theta v_z (v_e + v_\theta)) + 5mnR^4v_e^2v_\theta^2v_z^2 (v_e + v_\theta)^2 + 2R^6v_e^2v_\theta^3 (v_e + v_\theta)^3)(m^2v_\theta
\]
\[+ mnv_\theta + R^2v_e v_\theta (v_e + v_\theta))
\end{align*}
\]

and

\[
\begin{align*}
D^2 \frac{dH_0}{dn} &= 2m^3R^2v_e^2v_\theta v_z (m^2v_\theta + mnv_\theta + R^2v_e v_z (v_e + v_\theta))(m^4v_\theta^2
\]
\[+ 2m^2R^2v_e v_\theta v_z (v_e + v_\theta)) + R^4v_e^2v_\theta^2 (v_e + v_\theta)^2)
\end{align*}
\]

both of which are positive.

Behavior of the Derivatives of $H_1$ and $H_2$ with respect to $n$:

\[
\begin{align*}
D^2 \frac{dH_1}{dn} &= -m^3Rv_e v_\theta (m^2v_\theta + R^2v_e v_z (v_e + v_\theta))(m^5v_\theta^2
\]
\[+ 2m^4nv_\theta^2 + m^3v_\theta(n^2v_\theta + 2R^2v_e v_z (v_e + v_\theta)) + 2m^2nR^2v_\theta v_z (2v_e + v_\theta)
\end{align*}
\]

and is negative. Further,

\[
\begin{align*}
D^2 \frac{dH_2}{dn} &= 2m^2R^2v_e v_\theta v_z (m^2v_\theta + mnv_\theta + R^2v_e v_z (v_e + v_\theta))(m^4v_\theta^2
\]
\[+ 2m^3R^2v_e v_\theta v_z (v_e + v_\theta)) + R^4v_e^2v_\theta^2 (v_e + v_\theta)^2)
\end{align*}
\]

and is positive.

Behavior of the Derivatives of $H_1$ and $H_2$ with respect to $m$:

We find that

\[
\begin{align*}
D^2 \frac{dH_1}{dm} &= -Rv_e v_\theta (m^{10}v_\theta^5 + 2m^9nv_\theta^5 + m^8v_\theta^4(n^2v_\theta + 2R^2v_e v_z (v_e + 2v_\theta))
\]
\[+ 4m^7nR^2v_e v_\theta v_z (v_e + 2v_\theta)) + m^6R^2v_e v_\theta v_z (n^2v_\theta + 6v_\theta)
\end{align*}
\]

and

\[
\begin{align*}
D^2 \frac{dH_2}{dm} &= -Rv_e v_\theta (m^{10}v_\theta^5 + mnv_\theta + R^2v_e v_z (v_e + v_\theta))(m^8v_\theta^4 + 3m^7nv_\theta^4
\]
\[+ m^6v_\theta^3(3n^2v_\theta + R^2v_e v_z (v_e + 3v_\theta)) + m^5nv_\theta^3(n^2v_\theta + 2R^2v_e v_z (2v_e
\end{align*}
\]
The highest power of \( m \) in the numerator of the above derivatives are \(-m^{10}Rv_\varepsilon v_\theta^6\). Thus, for sufficiently large \( m \) the derivative is negative. As \( m \to 0 \), the derivatives go to \( \frac{v_\theta}{2Rv_\varepsilon v_\theta} \), which is positive.

**Behavior of the Derivatives of \( \text{var}(P) \) with respect to \( m, n, \) and \( v_\varepsilon \):** We have that

\[
D^2 \frac{d\text{var}(P)}{dm} = 2mR^2v_\varepsilon^2v_\varepsilon v_\varepsilon (2m^6v_\varepsilon^3 + 3m^5nv_\varepsilon^3 + 3m^4v_\varepsilon^2(n^2v_\varepsilon + 2R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)) \\
+ n^2v_\varepsilon^3 + 8R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta) + 3m^2R^2v_\varepsilon v_\theta v_\varepsilon (v_\varepsilon + v_\theta) \\
+ v_\varepsilon (n^2v_\varepsilon + 2R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)) + 5mvR^4v_\varepsilon^2v_\varepsilon^2(v_\varepsilon + v_\theta)^2 \\
+ 2R^6v_\varepsilon^4v_\varepsilon(v_\varepsilon + v_\theta)^3(m^2v_\varepsilon + mnv_\varepsilon + R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)) \\
+ m^2R^2v_\varepsilon v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta) + R^4v_\varepsilon^2v_\varepsilon^2(v_\varepsilon + v_\theta)^2),
\]

and

\[
D^2 \frac{d\text{var}(P)}{dv_\varepsilon} = m^2R^2v_\varepsilon^2v_\varepsilon v_\varepsilon (2m^6v_\varepsilon^3 + 4m^5nv_\varepsilon^3 + 3m^4v_\varepsilon^2(n^2v_\varepsilon + 2R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)) \\
+ 5mvR^4v_\varepsilon^2v_\varepsilon^2(v_\varepsilon + v_\theta)^2 \\
+ n^2v_\varepsilon^3 + 10R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta) + 3m^2R^2v_\varepsilon v_\theta v_\varepsilon (v_\varepsilon + v_\theta) \\
+ 2R^6v_\varepsilon^4v_\varepsilon(v_\varepsilon + v_\theta)^3(m^2v_\varepsilon + mnv_\varepsilon + R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)) \\
+ 2R^6v_\varepsilon^4v_\varepsilon(v_\varepsilon + v_\theta)^3(m^2v_\varepsilon + mnv_\varepsilon + R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)) \\
+ m^2R^2v_\varepsilon v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta) + R^4v_\varepsilon^2v_\varepsilon^2(v_\varepsilon + v_\theta)^2),
\]

The above derivatives are all positive.

**Proof of Proposition 2:** We have that

\[
\text{var}(\theta + \varepsilon | P) = v_\varepsilon + v_\theta - \frac{[\text{cov}(\theta, P)]^2}{\text{var}(P)}.
\]

Now, \( \text{cov}(\theta, P) = H_0v_\theta \)

and

\( \text{var}(P) = H_0^2v_\theta + (H_1^2 + H_2^2)v_\varepsilon. \)

Substituting for \( H_0, \ H_1, \) and \( H_2 \) from (7)-(9) yields (11). Let \( \gamma \equiv [\text{var}(\theta + \varepsilon | P)]^{-1}. \) We then have

\[
B^2 \frac{d\gamma}{dm} = 2mR^2v_\varepsilon^2v_\varepsilon (2m^8v_\varepsilon^4 + 5m^7nv_\varepsilon^4 + 2m^6v_\varepsilon^3(3n^2v_\varepsilon + 4R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)) \\
+ m^2v_\varepsilon^2 + 17R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)) + m^4v_\varepsilon^2(n^2v_\varepsilon^2 \\
+ 14n^2R^2v_\varepsilon v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta) + 12R^4v_\varepsilon^2v_\varepsilon^2(v_\varepsilon + v_\theta)^2).
\]
We also find that
\[ \frac{d \gamma}{dn} = \nu \]
where
\[ \nu = 2m^3 R^2 v_\gamma^3 v_z (m^2 v_\theta + mn v_\theta + R^2 v_\epsilon v_z (v_\epsilon + v_\theta)) \]
and
\[ \delta = (m^4 v_\theta^2 + 2m^3 n v_\theta^2 + m^2 v_\theta (n^2 v_\theta + 3R^2 v_\epsilon v_z (v_\epsilon + v_\theta))) + 2mnR^2 v_\gamma v_\theta v_z (v_\epsilon + v_\theta) \]
\[ + 2R^4 v_\epsilon^2 v_\theta^2 (v_\epsilon + v_\theta)^2. \]

The above derivatives are both positive. ||

**Proof of Proposition 3:** The first two parts of the proposition follow from (24) and (25) above. We also have that
\[ D^2 \frac{d(H_1 + H_2)}{dn} = -m^2 R v_\epsilon v_\theta^2 (m^2 v_\theta + R^2 v_\epsilon v_z (v_\epsilon + v_\theta)) (m^6 v_\theta + 2m^5 n v_\theta^3 \]
\[ + m^4 v_\theta (n^2 + 2R^2 v_\epsilon v_z) + 2m^3 n R^2 v_\epsilon v_\theta v_z (v_\epsilon + v_\theta) \]
\[ + m^2 R^2 v_\epsilon^2 v_\theta v_z (n^2 v_\theta + R^2 v_z (v_\epsilon + v_\theta)) (v_\theta - 3v_\epsilon) \]
\[ - 2R^6 v_\epsilon^4 v_z^3 (v_\epsilon + v_\theta)^2 \]
which is of ambiguous sign. ||

**Behavior of the Derivative in (26):** As \( n \to 0 \), the derivative in (26) tends to
\[ \frac{m^2 R v_\epsilon v_\theta^2 (2R^2 v_\epsilon^2 v_z - m^2 v_\theta) (m^2 v_\theta + R^2 v_\epsilon v_z (v_\epsilon + v_\theta))}{(m^2 v_\theta + 2R^2 v_\epsilon^2 v_z)^2 (m^2 v_\theta + R^2 v_\epsilon v_z (v_\epsilon + v_\theta))} \]
which is positive for small (but non-zero) \( m \). Note also that the derivative in (26) is negative for sufficiently large \( m \) and \( n \) because the derivative is preceded by a negative sign and the negative terms in brackets (which contribute positively to the derivative) do not involve \( m \) and \( n \).

**Proof of Proposition 4:** We denote the `\` \( A \) matrices'' in (13) by \( A_I \) and \( A_N \) for cash flow informed and noise informed traders, respectively. The wealth of the noise informed agents is given by
\[ \frac{\theta + \epsilon - P}{R v} = \frac{(\epsilon - H_0 \theta - H_1 z_1 - H_2 z_2 + \theta) (a_0 \theta + a_2 z_2 - H_0 \theta - H_1 z_1 - H_2 z_2)}{R v} \]
The matrix \( A_N \) then becomes
\[
\begin{bmatrix}
\frac{(1-H_0)(a_1 - H_0)}{2v} & a_1 - H_0 & \frac{H_1(a_1 - 2H_0 + 1)}{2v} & \frac{a_1H_2 + a_2(H_0 - 1) - H_2(2H_0 - 1)}{2v} \\
\frac{a_2 - H_2}{2v} & 0 & \frac{H_1}{2v} & \frac{a_2 - H_2}{2v} \\
\frac{-H_1(a_1 - 2H_0 + 1)}{2v} & \frac{H_1}{2v} & \frac{H_1^2}{v} & \frac{H_1(2H_2 - a_2)}{2v} \\
\frac{2v}{2v} & \frac{a_1H_2 + a_2(H_0 - 1) - H_2(2H_0 - 1)}{2v} & \frac{H_1(2H_2 - a_2)}{2v} & \frac{v}{v}
\end{bmatrix}
\]

Similarly, the wealth of cash flow informed agents is given by

\[
(\theta + \varepsilon - P)(\theta - P) = \frac{(\varepsilon - H_0 \theta - H_1 z_1 - H_2 z_2 + \theta)(H_0 \theta + H_1 z_1 + H_2 z_2 - \theta)}{R_{V_\varepsilon}}
\]

and the matrix \( A_I \) is

\[
\begin{bmatrix}
(H_0 - 1)^2 & 1 - H_0 & H_1(H_0 - 1) & H_2(H_0 - 1) \\
(H_0 - 1) & 1 - H_0 & H_1(H_0 - 1) & H_2(H_0 - 1) \\
(H_1 - 1) & 1 - H_0 & H_1(H_0 - 1) & H_2(H_0 - 1) \\
H_1 & 1 - H_0 & H_1(H_0 - 1) & H_2(H_0 - 1)
\end{bmatrix}
\]

Substituting for \( H_0, H_1, H_2, a_0, a_2, \) and \( \varepsilon, \) from (7)-(9), and (21)-(23), and then applying Lemma 1 completes the proof. ||

**Proof of Proposition 5:** We have that

\[
D^2 \frac{dDet_I}{dm} = -2m^2R^2v_\varepsilon v_\theta v_\varepsilon (m^2v_\theta + mnv_\theta + R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta))(m^2v_\theta + R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta))^2
\]

which is negative. Similarly

\[
D^2 \frac{dDet_N}{dm} = 2mv_\varepsilon v_\theta v_\varepsilon (1 - 2R)(m^2v_\theta + m^7nv_\theta + 2m^6R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta))
\]

\[
+ m^6nR^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta) + m^4R^2v_\varepsilon v_\theta v_\varepsilon (n^2v_\theta + R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)^2
\]

\[
+ m^3R^4v_\varepsilon v_\varepsilon v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)(v_\varepsilon - v_\theta) - 4m^2R^6v_\varepsilon v_\theta v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)^2
\]

\[
- mnR^6v_\varepsilon v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)^2 - 2R^8v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta)^4)
\]

and is of ambiguous sign. The third part of the proposition follows from comparing (11) and (14). This third part also implies since price efficiency is increasing in \( m \), the expected utility of trading on cash flow information is decreasing in \( m \). We also have that

\[
D^2 \frac{dDet_N}{dn} = -2m^3v_\varepsilon v_\theta v_\varepsilon (2R - 1)(m^2v_\theta + mnv_\theta + R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta))
\]

\[
+ v_\theta (m^2v_\theta + R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta))(m^2v_\theta + R^2v_\varepsilon v_\varepsilon (v_\varepsilon + v_\theta))
\]

which, given the required assumption for \( EU_N \) to exist, i.e., \( R > 0.5 \), is negative. ||

**Proof of Proposition 6:** A simple calculation shows that
\[ \alpha_d - \alpha = \frac{m^2 R^2 v_c^2 v_\theta^2 v_z [m^4 v_\theta^2 + 2m^2 R^2 v_c^2 v_\theta v_z (v_c + v_\theta) + R^4 v_c^2 v_z^2 (v_c + v_\theta)^2]}{2D(m^2 v_\theta + R^2 v_c^2 v_z)} > 0. \]

Since \( \alpha > \alpha_d \), the value of the private firm is maximized with disclosure. ||

**Proof of Proposition 8:** Suppose that \( \alpha_d + A_1 > C > \alpha + A_1 \), then suppose the firm commits to disclose. In this case, the competing bidder bids \( \alpha_d + A_1 \). The public is not able to match this bid and loses the surplus \( A - A_1 \). So disclosing is not an equilibrium. Now suppose the firm refrains from disclosing. Then, the competing firm's maximum bid is \( \alpha + A_1 \). The public firm is able to outbid. Hence not disclosing is an equilibrium. Now suppose that \( C > \alpha_d + A_1 \). If the public firm discloses, the competing bid is \( \alpha_d + A_1 \). If the firm does not disclose, the competing bid is \( \alpha + A_1 \). In either case the public firm pays a fair value for the cash flows of the private firm but captures the incremental synergistic surplus \( A - A_1 \). In other words, in either case, the net present value of the acquisition is \( A - A_1 \). So the firm is indifferent between disclosing or not disclosing \( z_1 \). And, if \( C < \alpha + A_1 \), then the firm is again indifferent between disclosing and not disclosing \( z_1 \) because when the cash balance is very low, the public firm able to outbid the competing bidder whether or not the signal \( z_1 \) is disclosed. ||